

Circular measure

A-Level Maths 9709 · Topic 1.4 revision notes

Everything from the 2 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Radians

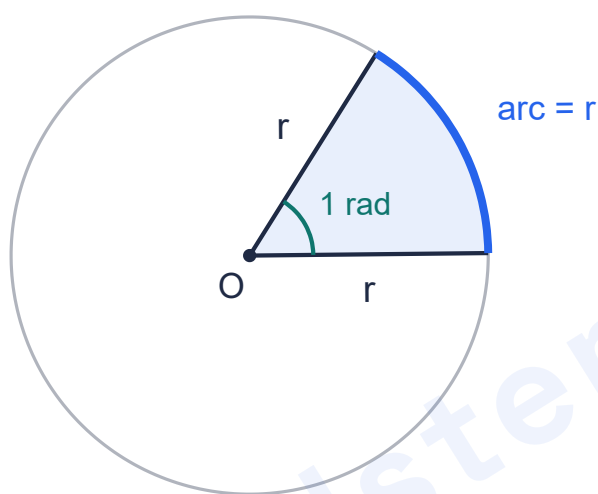
2. Arc length and sector area

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1 Radians

Syllabus 1.4

One radian is the angle at the centre of a circle subtended by an arc equal in length to the radius. A full turn is 2π radians, so $\pi \text{ rad} = 180^\circ$: multiply by $\frac{\pi}{180}$ to go from degrees to radians, and by $\frac{180}{\pi}$ to go back.



$$\pi \text{ rad} = 180^\circ$$

$$2\pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} \approx 57.3^\circ$$

Fig. 1: an arc equal in length to the radius r subtends an angle of 1 radian (about 57.3°) at the centre O

Worked example: Degrees to radians

Convert (a) 225° to radians, giving an exact answer, and (b) 50° to radians, giving your answer to 3 significant figures.

1. (a) Multiply by $\frac{\pi}{180}$: $225 \times \frac{\pi}{180} = \frac{225\pi}{180}$.

2. Cancel by 45: $\frac{225\pi}{180} = \frac{5\pi}{4}$.

3. (b) $50 \times \frac{\pi}{180} = \frac{5\pi}{18}$.

4. As a decimal, $\frac{5\pi}{18} = 0.8727\dots$

Answer: (a) $\frac{5\pi}{4}$ (b) 0.873 rad

Convert with $\pi = 180^\circ$,
then work in radian mode.

- ✓ **To radians:** multiply by $\frac{\pi}{180}$; leave the answer as a multiple of π when it is exact.
- ✓ **To degrees:** multiply by $\frac{180}{\pi}$; the π cancels.
- ✓ **Calculator:** radian mode for any angle without a degree sign.
- ✓ **Triangles:** $\frac{1}{2}ab \sin C$, sine rule and cosine rule as usual; the angles add to π .
- ✓ **Accuracy:** lengths to 3 s.f., radians to 3 s.f., degrees to 1 d.p.

Watch out!

Your calculator must be in the right mode. $\sin 1.2$ in degree mode gives 0.0209 instead of 0.932. If a question gives angles in radians (or uses π), switch to radian mode before pressing anything.

Do not multiply $\frac{7\pi}{12}$ by 180 and stop: that gives 105π , not 105° . The π in the angle cancels with the π in $\frac{180}{\pi}$.

Never mix units in an angle sum. If two angles are $\frac{\pi}{4}$ and 1.1, the third is $\pi - \frac{\pi}{4} - 1.1$, not $180 - \frac{\pi}{4} - 1.1$.

2 Arc length and sector area

Syllabus 1.4

For a sector of radius r and angle θ radians: arc length $s = r\theta$ and area $A = \frac{1}{2}r^2\theta$. A segment is a sector minus a triangle, so its area is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta$, and its chord is $2r \sin \frac{\theta}{2}$.

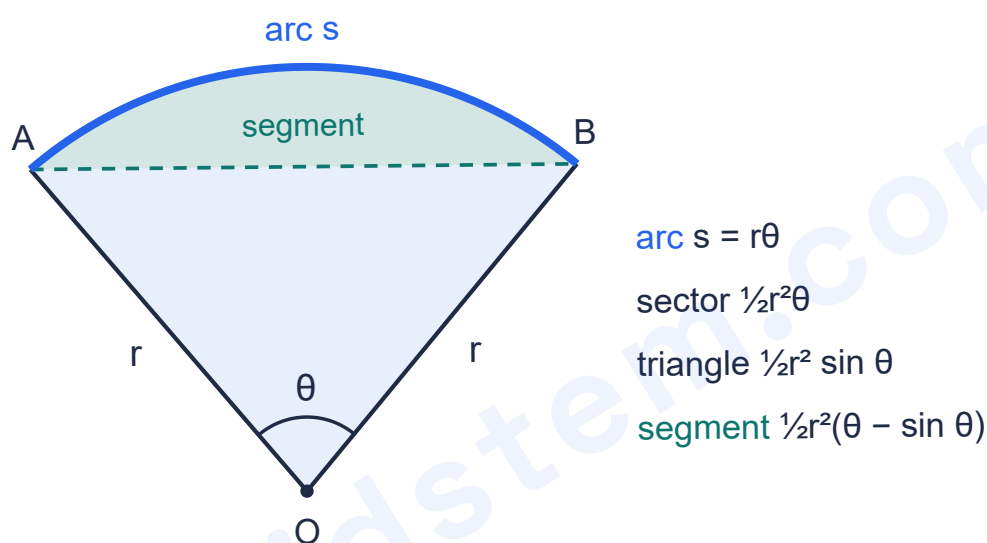


Fig. 1: sector OAB with angle θ radians. The shaded segment is the sector minus triangle OAB ; the dashed chord AB has length $2r \sin(\theta/2)$

Worked example: Arc and perimeter

A sector has radius 9 cm and angle 0.8 radians. Find the arc length and the perimeter of the sector.

1. Arc length $s = r\theta = 9 \times 0.8 = 7.2$ cm.
2. The perimeter is the arc plus two radii: $7.2 + 9 + 9$.

Answer: Arc = 7.2 cm, perimeter = 25.2 cm

Name the pieces, then use $r\theta$ and $\frac{1}{2}r^2\theta$.

- ✓ **Angle in radians:** convert first if it is given in degrees.
- ✓ **Arc:** $s = r\theta$. **Sector:** $A = \frac{1}{2}r^2\theta$.
- ✓ **Triangle:** $\frac{1}{2}r^2 \sin \theta$; **chord:** $2r \sin \frac{\theta}{2}$.
- ✓ **Tangents:** right angle with the radius, then right-angled trigonometry.
- ✓ **Compound regions:** add and subtract the simple pieces; perimeters use only the edges of the region.

Watch out!

$s = r\theta$ only works with θ in radians. If the angle is given as 60° , convert it to $\frac{\pi}{3}$ first; $9 \times 60 = 540$ cm is nonsense.

Forgetting the $\frac{1}{2}$ doubles the answer. A useful check: the sector must be smaller than the whole circle, πr^2 .

The triangle in a segment has area $\frac{1}{2}r^2 \sin \theta$, using the angle at the centre, not $\frac{1}{2} \times \text{chord} \times r$. And in a perimeter, use the straight chord, not another arc.