

Continuous random variables

A-Level Maths 9709 · Topic 6.3 revision notes

Everything from the 1 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Probability density functions, mean and variance

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1 Probability density functions, mean and variance

Syllabus 6.3

A probability density function satisfies $f(x) \geq 0$ and $\int f(x) dx = 1$ over its domain.

$$P(a < X < b) = \int_a^b f(x) dx, E(X) = \int xf(x) dx, \text{Var}(X) = \int x^2 f(x) dx - \mu^2, \text{ and the median } m \text{ has half the area to its left.}$$

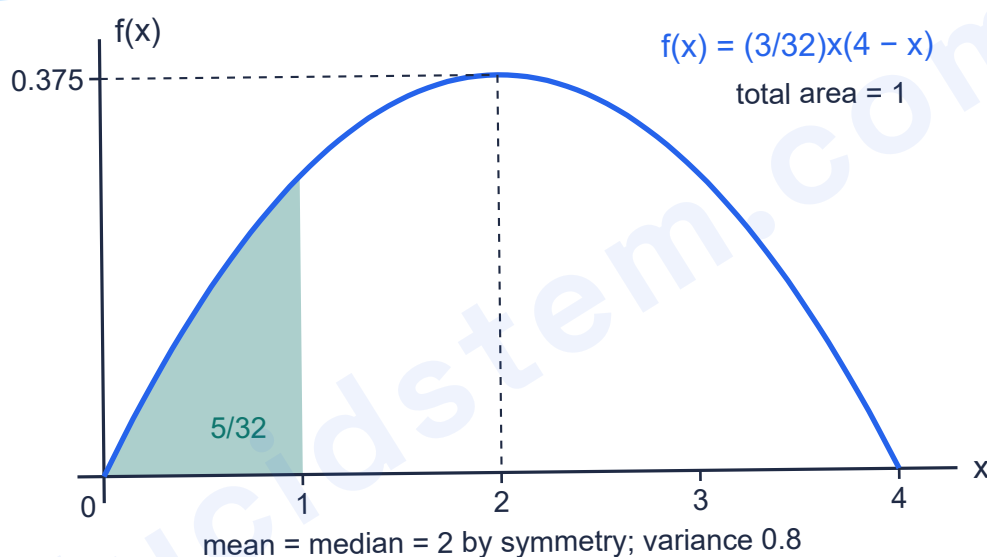


Fig. 1: the density $f(x) = (3/32)x(4-x)$ for $0 \leq x \leq 4$, drawn to scale. The total area is 1 and the shaded area is $P(X < 1) = 5/32$

Worked example: Finding the constant and a probability

X has density $f(x) = kx(4-x)$ for $0 \leq x \leq 4$, and 0 otherwise. Find k and $P(X < 1)$.

$$1. \int_0^4 (4x - x^2) dx = \left[2x^2 - \frac{x^3}{3} \right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}.$$

$$2. \text{Total area 1: } k \times \frac{32}{3} = 1, \text{ so } k = \frac{3}{32}.$$

$$3. P(X < 1) = \frac{3}{32} \left[2x^2 - \frac{x^3}{3} \right]_0^1 = \frac{3}{32} \times \frac{5}{3}.$$

$$4. = \frac{5}{32} = 0.15625 \text{ (Fig. 1).}$$

Answer: $k = \frac{3}{32}$, $P(X < 1) = \frac{5}{32} = 0.156$

Total area 1, then integrate what you need.

- ✓ **Unknown constant:** set the integral over the whole domain equal to 1.
- ✓ **Probability:** integrate f over the interval; end points do not matter.
- ✓ **Mean:** $\int x f(x) dx$, or use symmetry and say so.
- ✓ **Variance:** $\int x^2 f(x) dx - \mu^2$.
- ✓ **Median or percentile:** area to the left equals 0.5 (or the percentage); reject roots outside the domain.
- ✓ **Infinite domain:** use $x^{-n} \rightarrow 0$ as $x \rightarrow \infty$.

Watch out!

$f(2) = \frac{3}{8}$ is not $P(X = 2)$. For a continuous variable that probability is 0; a value of f only becomes a probability after you integrate it over an interval.

$\text{Var}(X) = \int x^2 f(x) dx$ is NOT the variance until you subtract μ^2 . Leaving the answer as 4.8 above is the most frequent lost mark.

The median is not half-way along the domain. For $f(x) = \frac{x}{8}$ on $[0, 4]$ it is 2.83, not 2: more of the area sits on the right where the density is higher.