

Coordinate geometry

A-Level Maths 9709 · Topic 1.3 revision notes

Everything from the 3 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Equations of straight lines

2. The equation of a circle

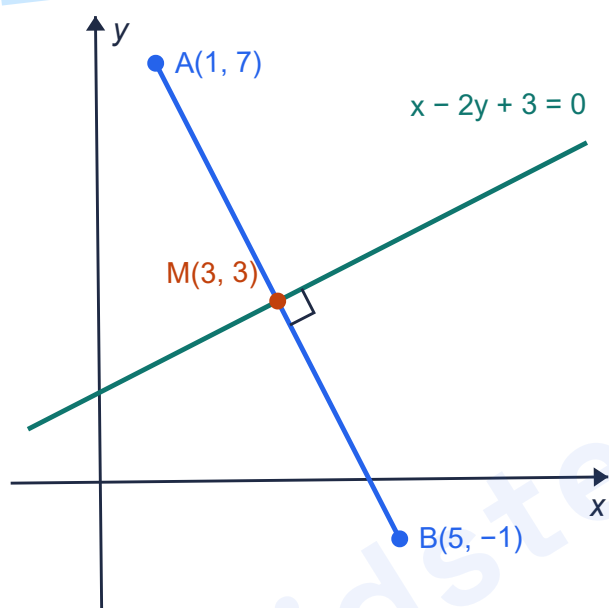
3. Intersections of lines and curves

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1 Equations of straight lines

Syllabus 1.3

The line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$. Parallel lines have equal gradients; perpendicular lines have $m_1 m_2 = -1$, so a perpendicular gradient is the negative reciprocal $-\frac{1}{m}$.



$$\text{gradient } AB = -8 \div 4 = -2$$

$$\text{perpendicular gradient} = \frac{1}{2}$$

$$(-2) \times \frac{1}{2} = -1$$

line through $M(3, 3)$:

$$y - 3 = \frac{1}{2}(x - 3)$$

Fig. 1: the perpendicular bisector of $A(1, 7)$ and $B(5, -1)$ passes through the midpoint $M(3, 3)$ with gradient $\frac{1}{2}$, because AB has gradient -2

Worked example: Through two points

Find the equation of the line through $A(-2, 5)$ and $B(4, -1)$, in the form $ax + by + c = 0$.

$$1. m = \frac{-1 - 5}{4 - (-2)} = \frac{-6}{6} = -1.$$

$$2. \text{ Using } A: y - 5 = -1(x + 2) \Rightarrow y = -x + 3.$$

$$3. \text{ Check with } B: -4 + 3 = -1, \text{ as required. Rearrange so everything is on one side.}$$

Answer: $x + y - 3 = 0$

Find a gradient and a point, then write the line.

- ✓ **Gradient** from two points, or by rearranging a given line to $y = mx + c$.
- ✓ **Parallel**: same gradient. **Perpendicular**: $-\frac{1}{m}$.
- ✓ **Equation**: $y - y_1 = m(x - x_1)$, then rearrange to the form asked for.
- ✓ **Length and midpoint** from the formulae; **intersections** by solving simultaneously.
- ✓ **Sketch** the points to check that your answers are sensible.

Watch out!

The gradient of $3x + 4y = 24$ is not 3 (nor $\frac{4}{3}$). Always rearrange to $y = mx + c$ before reading m .

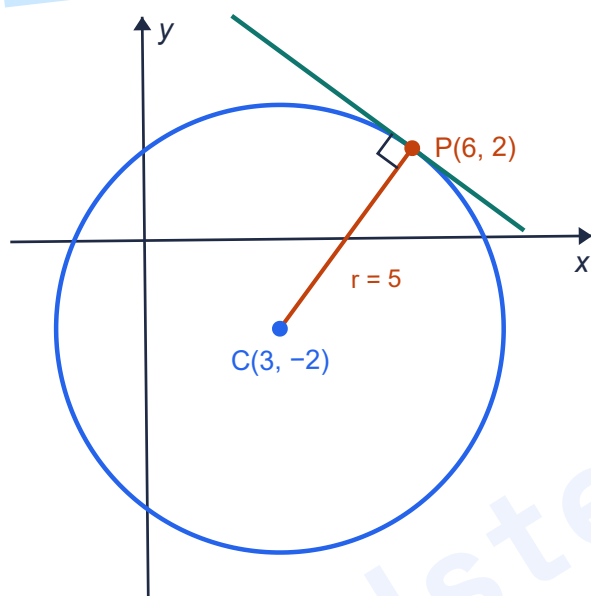
The perpendicular gradient is the **NEGATIVE** reciprocal. For $m = -2$ it is $+\frac{1}{2}$; $-\frac{1}{2}$ (reciprocal only) and 2 (sign change only) are both wrong.

In the length formula, subtract the coordinates before squaring: from $(-1, 3)$ to $(5, -5)$ the changes are 6 and -8 , giving $\sqrt{36 + 64} = 10$. Adding the coordinates (4 and -2) is a common slip.

2 The equation of a circle

Syllabus 1.3

$(x - a)^2 + (y - b)^2 = r^2$ is the circle with centre (a, b) and radius r . For $x^2 + y^2 + 2gx + 2fy + c = 0$, complete the square: the centre is $(-g, -f)$ and the radius is $\sqrt{g^2 + f^2 - c}$.



$$(x - 3)^2 + (y + 2)^2 = 25$$

radius CP: gradient $\frac{4}{3}$

tangent at P: gradient $-\frac{3}{4}$

$$3x + 4y - 26 = 0$$

Fig. 1: the circle with centre $C(3, -2)$ and radius 5 passes through $P(6, 2)$; the tangent at P is perpendicular to the radius CP

Worked example: Write the equation

Find the equation of the circle with centre $(3, -2)$ and radius 5, and show that $P(6, 2)$ lies on it (Fig. 1).

1. $a = 3$, $b = -2$, $r = 5$: $(x - 3)^2 + (y - (-2))^2 = 5^2$.

2. So $(x - 3)^2 + (y + 2)^2 = 25$.

3. At P : $(6 - 3)^2 + (2 + 2)^2 = 9 + 16 = 25$, so P is on the circle.

Answer: $(x - 3)^2 + (y + 2)^2 = 25$; P lies on it

Get to $(x - a)^2 + (y - b)^2 = r^2$.

- ✓ **Centre and radius** give the equation directly; remember r^2 on the right.
- ✓ **Expanded form**: make the x^2 and y^2 coefficients 1, then complete both squares.
- ✓ **Diameter**: centre = midpoint, radius = half the length.
- ✓ **Tangent** at P : perpendicular to the radius CP .
- ✓ **Inside or outside**: compare the distance from the centre with r .

Watch out!

The right-hand side is r^2 , not r : a circle of radius 5 has $= 25$, and $(x - 3)^2 + (y + 2)^2 = 12$ has radius $\sqrt{12} = 2\sqrt{3}$, not 12.

The coefficients of x^2 and y^2 must be 1 before you read off g and f . For $2x^2 + 2y^2 - 8x + 12y - 6 = 0$, divide by 2 first. And watch the signs: $-6x$ gives centre $x = +3$.

The tangent's gradient is the negative reciprocal of the RADIUS gradient, not of the gradient from the origin to P . Always use the centre, which is only the origin for $x^2 + y^2 = r^2$.

3 Intersections of lines and curves

Syllabus 1.3

Substitute the line into the curve to get a quadratic. Its discriminant decides the picture: $b^2 - 4ac > 0$, the line cuts the curve at two points; $= 0$, the line is a tangent; < 0 , the line does not meet the curve.

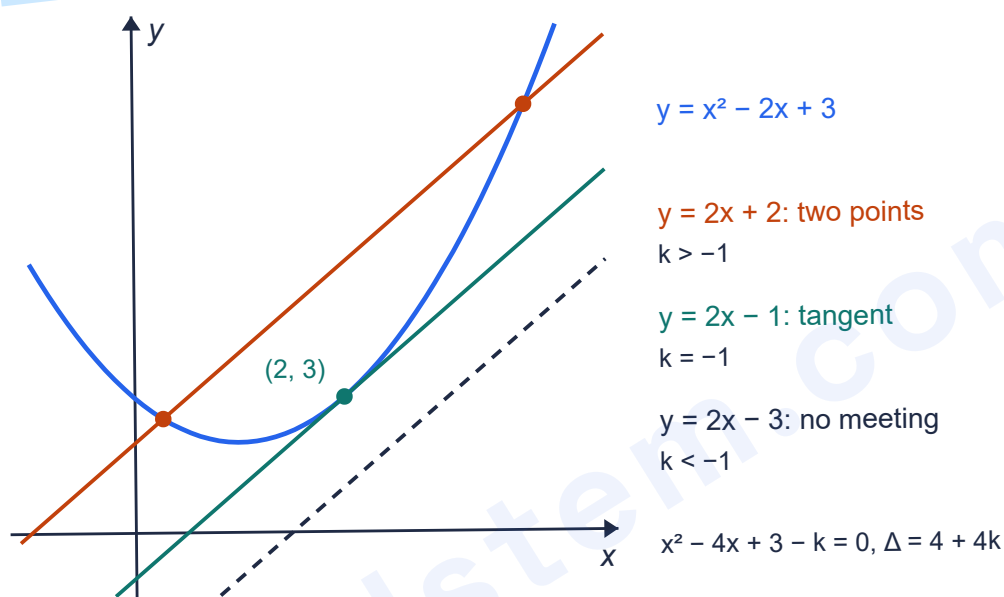


Fig. 1: lines $y = 2x + k$ against the curve $y = x^2 - 2x + 3$. Substituting gives $x^2 - 4x + 3 - k = 0$ with discriminant $4 + 4k$, so $k = -1$ gives the tangent at $(2, 3)$

Worked example: Line meets parabola

Find the points where the line $y = x - 1$ meets the curve $y = x^2 - 3x + 2$.

1. At an intersection both equations hold: $x^2 - 3x + 2 = x - 1$.
2. $x^2 - 4x + 3 = 0 \Rightarrow (x - 1)(x - 3) = 0$.
3. $x = 1$: $y = 0$. $x = 3$: $y = 2$.

Answer: $(1, 0)$ and $(3, 2)$

One equation, one quadratic, one discriminant.

- ✓ **Substitute** the line into the curve or circle.
- ✓ **Collect** everything on one side: $ax^2 + bx + c = 0$, where a, b, c may contain the constant.
- ✓ **Decide**: two points > 0 ; tangent $= 0$; no meeting < 0 .
- ✓ **Solve** for the constant, then find any point of contact from the repeated root.
- ✓ **State** the conclusion in words, e.g. 'so the line is a tangent'.

Watch out!

Points of intersection need both coordinates. After finding x , substitute into the LINEAR equation for y (it is simpler and avoids pairing errors).

Collect EVERYTHING on one side before reading a, b, c : in $x^2 + (2 - m)x + 4 = 0$ the coefficient b is the whole bracket $(2 - m)$, not 2 and not $-m$.

Showing a line is a tangent needs a stated conclusion: ' $b^2 - 4ac = 0$, so there is a repeated root, so the line is a tangent'. Finding a single root without saying why it means tangency loses the explanation mark.