

Energy, work and power

A-Level Maths 9709 · Topic 4.5 revision notes

Everything from the 3 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Work done by a force

2. Kinetic and potential energy, the work-energy principle

3. Power and driving force

thelucidstem.com

1 Work done by a force

Syllabus 4.5

Work done by a constant force $= Fd \cos \theta$, where d is the distance moved and θ the angle between the force and the direction of motion. Work is measured in joules ($1 \text{ J} = 1 \text{ N m}$). Work against gravity $= mgh$; work against friction $= Fd$ with $F = \mu R$.

$$W = 60 \times 8 \times \cos 25^\circ = 435 \text{ J}$$

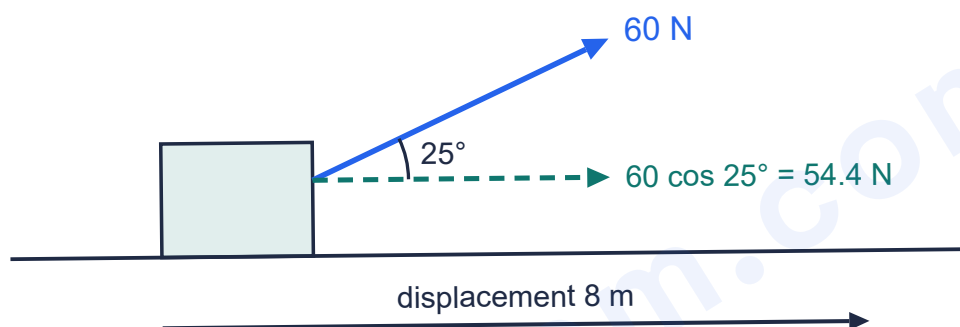


Fig. 1: a 60 N pull at 25° to the horizontal moves a crate 8 m. Only the 54.4 N component along the motion does work (angle drawn to scale)

Worked example: A rope at an angle (Fig. 1)

A crate is pulled 8 m along a horizontal floor by a rope with tension 60 N inclined at 25° above the horizontal. Find the work done by the tension.

1. Component of the tension along the motion $= 60 \cos 25^\circ = 54.38 \text{ N}$.
2. $W = 54.38 \times 8$.

Answer: 435 J (3 s.f.)

Resolve along the motion, times distance.

- ✓ **Work** = $Fd \cos \theta$, θ between force and motion.
- ✓ **Perpendicular forces** (normal reaction on a level floor) do no work.
- ✓ **Gravity**: $mg \times$ vertical height.
- ✓ **Friction**: find R first, then $\mu R \times d$.
- ✓ **Constant speed**: work done by the driving force = total work done against the resistances.

Watch out!

Do not use the full force when it is at an angle: $60 \times 8 = 480 \text{ J}$ is the work only if the rope is horizontal.

Work done by the weight on a slope uses the VERTICAL height, $mg \times d \sin \alpha$, not $mg \times d$. The distance along the slope only multiplies the component $mg \sin \alpha$.

With the rope at an angle, the normal reaction is NOT mg . Using μmg for friction in the first example gives 800 J instead of 650 J .

2 Kinetic and potential energy, the work-energy principle

Syllabus 4.5

$KE = \frac{1}{2}mv^2$ and gravitational $PE = mgh$. If only gravity does work, $KE + PE$ is constant. Otherwise use the work-energy principle: initial energy + work done by driving forces = final energy + work done against resistances.

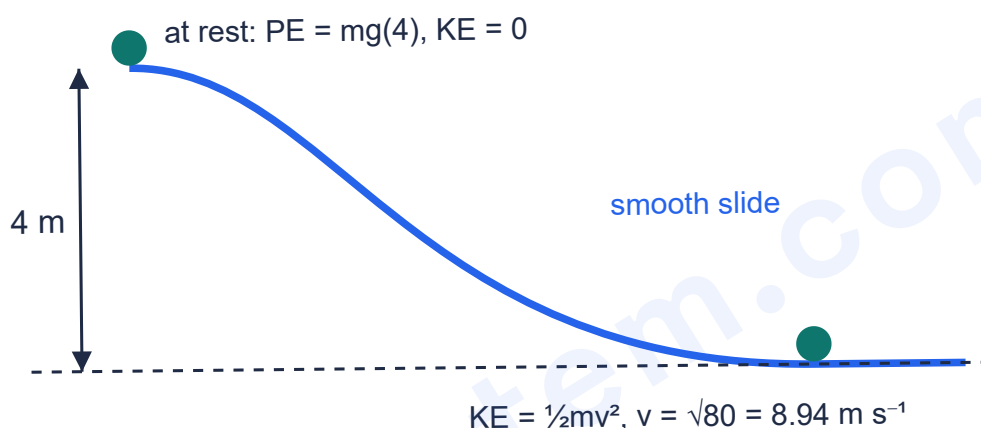


Fig. 1: a child slides from rest down a smooth curved slide that drops 4 m. Only gravity does work, so $mg(4) = \frac{1}{2}mv^2$ whatever the shape of the slide

Worked example: A smooth curved slide (Fig. 1)

A child of mass 30 kg starts from rest at the top of a smooth slide and descends a vertical height of 4 m. Find the speed at the bottom.

1. $PE \text{ lost} = 30 \times 10 \times 4 = 1200 \text{ J}$.
2. $KE \text{ gained: } \frac{1}{2}(30)v^2 = 1200$, so $v^2 = 80$.
3. $v = 8.94 \text{ m s}^{-1}$.

Answer: 8.94 m s^{-1} . The mass cancels: $v = \sqrt{2gh}$.

Start + work in = end + work out.

- ✓ **KE** = $\frac{1}{2}mv^2$ at the start and the end.
- ✓ **PE** change = $mg \times$ vertical height change (not distance along a slope).
- ✓ **Work in:** driving forces. **Work out:** friction and resistance ($\mu R \times d$).
- ✓ **Equation:** initial KE + PE + work in = final KE + PE + work out.
- ✓ **Curved paths:** energy is the only method you need.

Watch out!

Adding speeds instead of energies. The ball thrown at 12 and gaining 10 from the fall does NOT land at 22: energies add, so $v^2 = 144 + 100$.

$\frac{1}{2}m(25 - 15)^2$ is NOT the change in KE. Square each speed first: $\frac{1}{2}m(25^2 - 15^2)$.

Watch the sign of the KE change. Here the cyclist SLOWS down, so the lost KE (1560 J) helps:
 $W = 18\,000 - 1560$, not $18\,000 + 1560$.

3 Power and driving force

Syllabus 4.5

Power = $\frac{\text{work done}}{\text{time taken}}$, in watts ($1 \text{ W} = 1 \text{ J s}^{-1}$). For a force in the direction of motion,

$P = Fv$, so the driving force is $F = \frac{P}{v}$. At maximum speed the acceleration is zero and the driving force equals the total resistance.

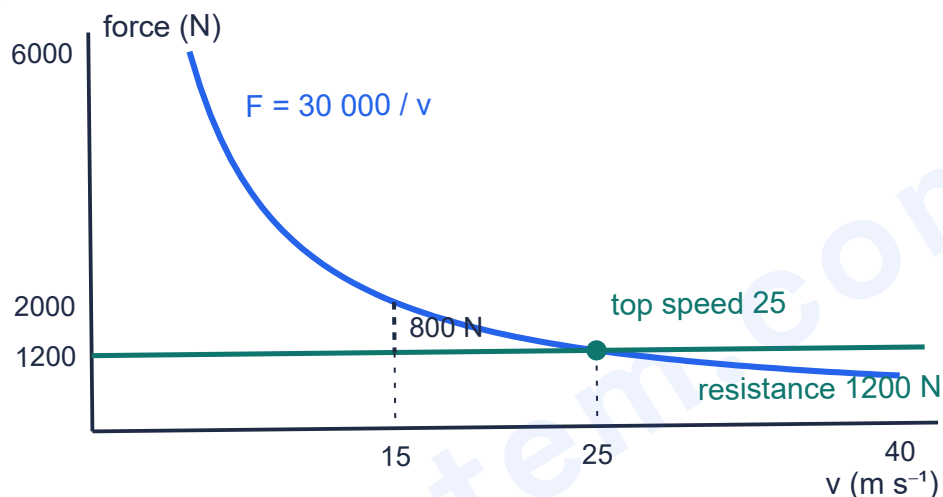


Fig. 1: driving force $F = P/v$ for a 30 kW engine (to scale) against a constant 1200 N resistance. At 15 m s^{-1} there is an 800 N resultant; at 25 m s^{-1} the forces balance and the car can go no faster

Worked example: Average power

A motor lifts a load of mass 50 kg through a vertical height of 12 m at constant speed in 8 s. Find the power of the motor.

1. Work done against gravity = $50 \times 10 \times 12 = 6000 \text{ J}$.

2. Power = $\frac{6000}{8}$.

Answer: 750 W

$$F = P/v, \text{ then } F = ma.$$

- ✓ **Units:** power in watts ($\text{kW} \times 1000$).
- ✓ **Driving force:** $F = \frac{P}{v}$ at the speed in question.
- ✓ **Hill:** $mg \sin \alpha$ opposes motion uphill and helps downhill.
- ✓ **Newton's second law:** driving force — resistances = ma .
- ✓ **Top speed:** set $a = 0$.

Watch out!

Power must be in WATTS before you use $F = P/v$. An engine of 30 kW gives $F = 30\,000/v$, not $30/v$.

$P = Fv$ uses the DRIVING force. Writing $P = mav$ (the resultant) at 15 m/s gives $a = 1.67$ instead of 0.667.

Downhill the weight component is on the SAME side as the driving force. Adding it to the resistance gives a top speed of 21.4 m/s for the lorry instead of 50 m/s.