

Forces and equilibrium

A-Level Maths 9709 · Topic 4.1 revision notes

Everything from the 3 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Forces, components and resultants

2. Equilibrium of a particle

3. Friction and limiting equilibrium

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1 Forces, components and resultants

Syllabus 4.1

A force F at angle θ to a direction has component $F \cos \theta$ along that direction and $F \sin \theta$ perpendicular to it. To find a resultant, add all the components in two perpendicular directions to get X and Y ; then $R = \sqrt{X^2 + Y^2}$ and $\tan \alpha = \frac{Y}{X}$.

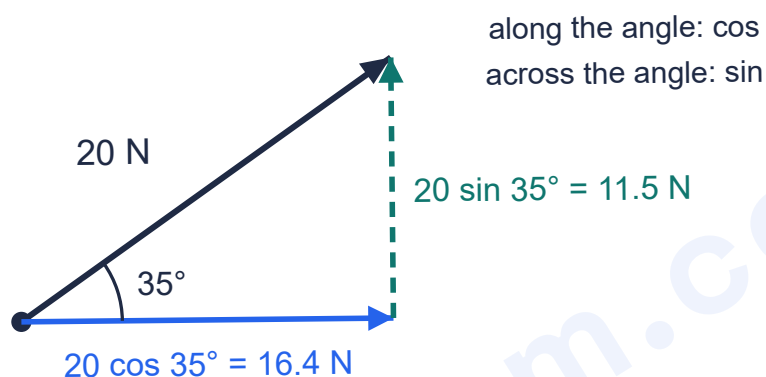


Fig. 1: a 20 N force at 35° to the horizontal splits into 16.4 N horizontally and 11.5 N vertically (drawn to scale)

Worked example: Horizontal and vertical components

A force of 50 N acts at 30° above the horizontal. Find its horizontal and vertical components.

1. The horizontal direction is next to the 30° angle, so use $\cos$: $50 \cos 30^\circ = 43.3 \text{ N}$.
2. The vertical direction is across from the angle, so use $\sin$: $50 \sin 30^\circ = 25 \text{ N}$.

Answer: 43.3 N horizontally and 25 N vertically upwards.

Two directions, signs, then Pythagoras.

- ✓ **Draw:** a force diagram with every force labelled, weight as mg with $g = 10$.
- ✓ **Choose directions:** horizontal/vertical, or along/perpendicular to a slope.
- ✓ **Resolve:** next to the angle is $\cos$, across from it is $\sin$.
- ✓ **Add with signs:** X and Y , negative for left/down.
- ✓ **Rebuild:** $R = \sqrt{X^2 + Y^2}$, $\tan \alpha = Y/X$, and say which way it points.

Watch out!

Weight and normal reaction are NOT a third-law pair: they act on the same body and are different kinds of force. A third-law pair always acts on two different bodies and is the same type of force.

Do not learn "horizontal is $\cos$ ". The rule is: the component next to the angle you are given uses $\cos$. If the angle is measured from the vertical, the vertical component is the $\cos$ one.

Keep the signs. A force pointing left or down gives a NEGATIVE component; adding its size instead is the most common way to lose every mark in a resultant question.

2 Equilibrium of a particle

Syllabus 4.1

A particle is in equilibrium when the vector sum of the forces on it is zero. Equivalently, the sum of the components in ANY direction is zero, so resolving in two perpendicular directions gives two equations for two unknowns.

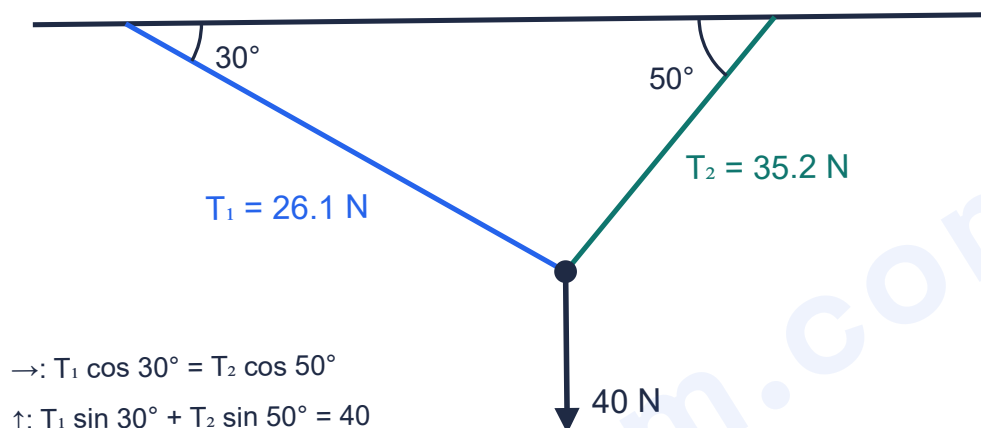


Fig. 1: a 40 N weight hanging from two strings at 30° and 50° to the horizontal (drawn to scale); resolving gives tensions of 26.1 N and 35.2 N

Worked example: A weight on two strings

A particle of weight 40 N hangs in equilibrium from two light strings inclined at 30° and 50° to the horizontal (Fig. 1). Find the tensions.

1. Horizontally: $T_1 \cos 30^\circ = T_2 \cos 50^\circ \dots (1)$
2. Vertically: $T_1 \sin 30^\circ + T_2 \sin 50^\circ = 40 \dots (2)$
3. From (1): $T_1 = \frac{T_2 \cos 50^\circ}{\cos 30^\circ} = 0.7422 T_2$.
4. Substitute in (2): $0.3711 T_2 + 0.7660 T_2 = 40$, so $T_2 = 35.2 \text{ N}$.
5. $T_1 = 0.7422 \times 35.18 = 26.1 \text{ N}$.

Answer: $T_1 = 26.1 \text{ N}$, $T_2 = 35.2 \text{ N}$. The steeper string carries more tension.

Force diagram, resolve twice, solve.

- ✓ **Diagram:** every force, weight mg with $g = 10$.
- ✓ **Directions:** horizontal/vertical for strings; parallel/perpendicular for slopes.
- ✓ **Equations:** sum of components $= 0$ in each direction.
- ✓ **Solve:** substitute, or square-and-add and divide for a force of unknown size and direction.
- ✓ **Check:** steeper strings carry more tension; answers to 3 s.f., angles to 1 d.p.

Watch out!

Keep at least 4 significant figures in the middle of the working.

Rounding T_2 to 35 before finding T_1 gives 26.0 instead of 26.1 and can cost the accuracy mark.

The normal reaction is NOT always $mg \cos \theta$. Any other force with a component perpendicular to the plane changes it. Always write the full perpendicular equation.

Dividing the two equations gives $\tan \theta$ only if you put the right

one on top: $\frac{P \sin \theta}{P \cos \theta} = \tan \theta$.

Check which component used sin before you divide.

3 Friction and limiting equilibrium

Syllabus 4.1

A rough contact gives a normal reaction R and a friction force F , with $F \leq \mu R$. While the particle stays at rest, F is only as large as needed. In limiting equilibrium ("about to slip") $F = \mu R$, acting against the direction the particle is about to move.

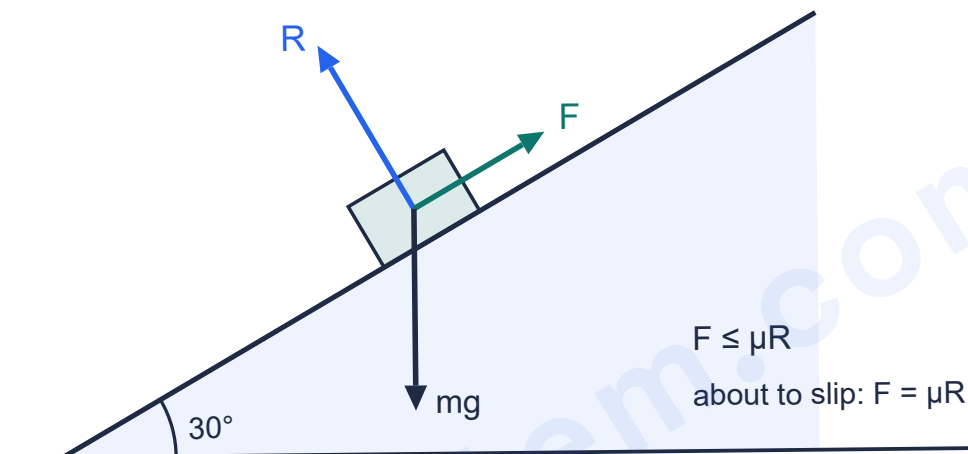


Fig. 1: a particle at rest on a rough plane inclined at 30° . The normal reaction R is perpendicular to the plane; friction F acts up the plane, against the direction the particle would slide

Worked example: Does it move?

A box of mass 8 kg rests on rough horizontal ground, $\mu = 0.4$. A horizontal force of 20 N is applied. Does the box move? Find the friction force.

1. Vertically: $R = 8 \times 10 = 80 \text{ N}$.
2. Maximum friction $= \mu R = 0.4 \times 80 = 32 \text{ N}$.
3. The pull, 20 N , is less than 32 N , so friction can balance it and the box stays at rest.
4. Horizontally: $F = 20 \text{ N}$.

Answer: The box does not move; the friction force is 20 N (not 32 N).

Find R first, then decide about F .

- ✓ **Diagram:** weight, normal reaction, friction and any applied forces.
- ✓ **Perpendicular to the surface:** find R , including components of applied forces.
- ✓ **Direction of friction:** opposite to the way the particle moves or is about to move.
- ✓ **Limiting?** If moving or about to slip, $F = \mu R$; otherwise $F \leq \mu R$.
- ✓ **Parallel to the surface:** resolve and solve.

Watch out!

Never write $F = \mu R$ automatically. It is only true when the particle is moving or about to slip. A box at rest with a small push has friction equal to the push.

Using $R = mg$ when the applied force has a vertical component. For the 5 kg block this gives $\mu = 27.19 \div 50 = 0.544$, a very common wrong answer.

Friction changes direction between "about to slip down" and "about to slip up". Using the same sign for both gives one answer twice instead of a range.