

Kinematics of motion in a straight line

A-Level Maths 9709 · Topic 4.2 revision notes

Everything from the 3 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Displacement, velocity and acceleration graphs

2. Constant acceleration: the suvat equations

3. Variable acceleration: using calculus

1 Displacement, velocity and acceleration graphs

Syllabus 4.2

Gradient of a displacement-time graph = velocity. Gradient of a velocity-time graph = acceleration. Area between a velocity-time graph and the time axis = displacement, with area below the axis counting as negative displacement.

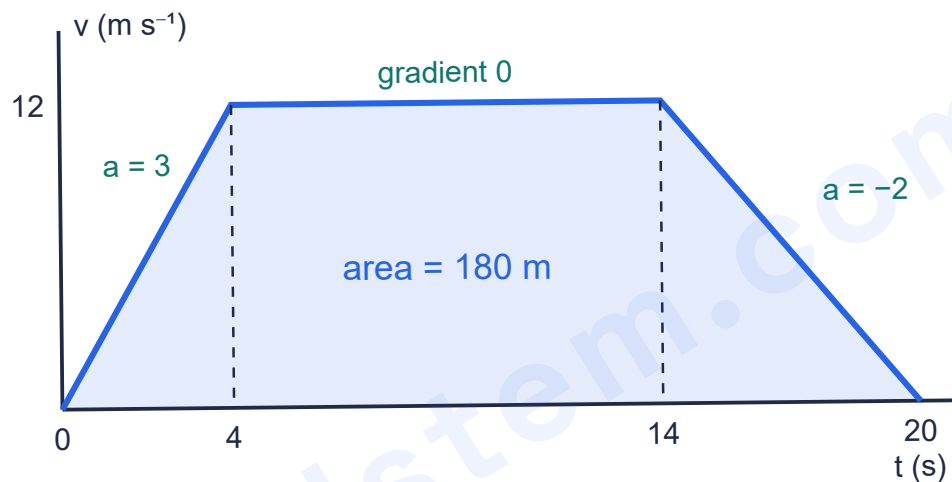


Fig. 1: velocity-time graph of a journey (to scale). Gradients give accelerations of 3, 0 and -2 m s^{-2} ; the shaded area is the 180 m travelled

Worked example: Distance versus displacement

A particle moves 5 m east and then 8 m west, taking 6.5 s in total. Find its distance travelled, displacement, average speed and average velocity.

1. Distance = $5 + 8 = 13 \text{ m}$.
2. Taking east as positive, displacement = $5 - 8 = -3 \text{ m}$, i.e. 3 m west.
3. Average speed = $\frac{13}{6.5} = 2 \text{ m s}^{-1}$.
4. Average velocity = $\frac{-3}{6.5} = -0.462 \text{ m s}^{-1}$.

Answer: 13 m; 3 m west; 2 m s^{-1} ; 0.462 m s^{-1} west.

Sketch it, then gradients and areas.

- ✓ **Sketch** the velocity-time graph, marking every time and velocity you know.
- ✓ **Gradient** of $v-t$ is acceleration; of $s-t$ is velocity.
- ✓ **Area** under $v-t$ is displacement: triangles, rectangles, trapezia.
- ✓ **Sign:** area below the axis is negative displacement but adds to distance.
- ✓ **Two particles:** write each displacement in terms of t and set them equal.

Watch out!

Average speed is total DISTANCE divided by time, not the average of the start and end speeds (that only works for constant acceleration).

On a displacement-time graph the HEIGHT is position, not speed. A particle can be far from the start (high on the graph) and still be at rest (flat line).

Area below the time axis is negative DISPLACEMENT but positive DISTANCE. Read the question: "distance travelled" adds the areas; "displacement" subtracts.

2 Constant acceleration: the suvat equations

Syllabus 4.2

For constant acceleration in a straight line: $v = u + at$, $s = \frac{1}{2}(u + v)t$, $s = ut + \frac{1}{2}at^2$, $s = vt - \frac{1}{2}at^2$, $v^2 = u^2 + 2as$. Each one leaves out one of the five quantities, so pick the equation without the one you neither know nor want.

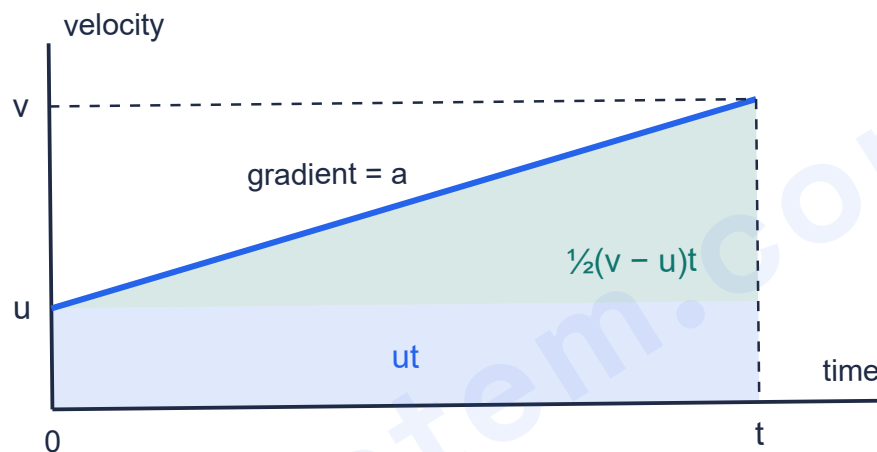


Fig. 1: with constant acceleration the velocity-time graph is a straight line. Gradient a gives $v = u + at$; the area $ut + \frac{1}{2}(v - u)t = \frac{1}{2}(u + v)t$ is the displacement

Worked example: Choosing the equation

A car travelling at 25 m s^{-1} brakes uniformly and stops in 60 m . Find the deceleration and the time taken.

1. Known: $u = 25$, $v = 0$, $s = 60$. Want a , not t : use $v^2 = u^2 + 2as$.
2. $0 = 625 + 120a$, so $a = -5.21 \text{ m s}^{-2}$.
3. For t , avoid the rounded a : $s = \frac{1}{2}(u + v)t$ gives $60 = 12.5t$.

Answer: Deceleration 5.21 m s^{-2} ; time 4.8 s .

List s , u , v , a , t , then choose.

- ✓ **Positive direction:** choose it and give every vector a sign.
- ✓ **List** s , u , v , a , t : which three do you know, which one do you want?
- ✓ **Choose** the equation that leaves out the fifth.
- ✓ **Gravity:** $a = -10$ with up positive, all the way up and down.
- ✓ **Stages:** the end of one stage is the start of the next.

Watch out!

The suvat equations need **CONSTANT** acceleration. If a question gives a or v as a function of t , use calculus instead (next lesson).

Using $s = +2$ for the drop to the ground. Displacement is measured from the point of projection, so landing below it means a **NEGATIVE** displacement when up is positive.

Restart the clock carefully. If B is released 1 s after A, then B has been moving for $(t - 1)$ s when A has been moving for t s.

3 Variable acceleration: using calculus

Syllabus 4.2

$v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$. Going back, $v = \int a \, dt$ and $s = \int v \, dt$, each with a constant found from a known value (usually at $t = 0$). Distance travelled is the total area between the velocity graph and the axis, so split the integral wherever $v = 0$.

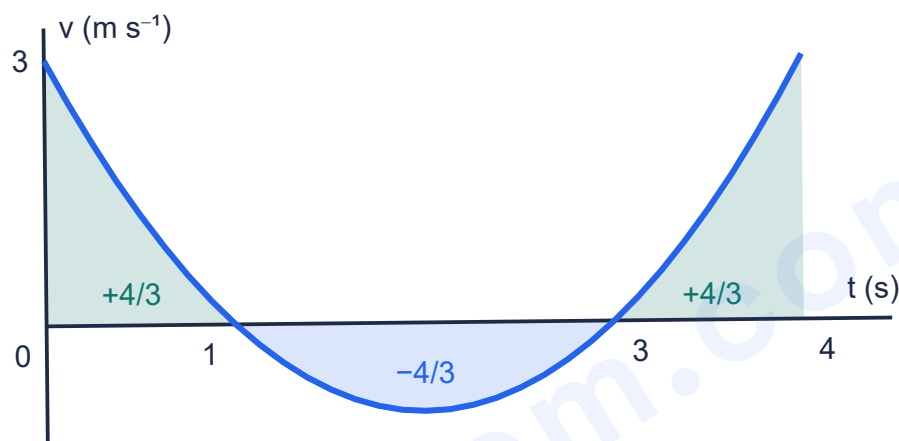


Fig. 1: $v = t^2 - 4t + 3$ for $0 \leq t \leq 4$ (to scale). The particle reverses at $t = 1$ and $t = 3$. Displacement $= \frac{4}{3} - \frac{4}{3} + \frac{4}{3} = \frac{4}{3}$ m, but distance $= \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4$ m

Worked example: Times at rest

A particle moves in a straight line with displacement $s = t^3 - 6t^2 + 9t$ metres from 0 at time t s. Find when it is at rest, and its acceleration at those times.

1. $v = \frac{ds}{dt} = 3t^2 - 12t + 9 = 3(t - 1)(t - 3)$.
2. $v = 0$ when $t = 1$ and $t = 3$.
3. $a = \frac{dv}{dt} = 6t - 12$: at $t = 1$, $a = -6$; at $t = 3$, $a = 6$.

Answer: At rest at $t = 1$ ($a = -6 \text{ m s}^{-2}$) and $t = 3$ ($a = 6 \text{ m s}^{-2}$).

Down by differentiating, up by integrating.

- ✓ **Down:** $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$.
- ✓ **Up:** integrate, then use a known value to find the constant.
- ✓ **At rest:** $v = 0$. **Maximum velocity:** $a = 0$.
- ✓ **Distance:** find where $v = 0$, split, and add the sizes.
- ✓ **Stages:** the velocity at the end of one formula starts the next.

Watch out!

$s(4) - s(0)$ is the **DISPLACEMENT**. If the particle turns round in the interval, the distance is bigger: find every time when $v = 0$ first.

Forgetting $+c$. If the particle starts with $v = 3$, dropping the constant gives the wrong velocity at every later time and every answer after it.

Integrating straight from 0 to 4 gives $\frac{4}{3}$, because the negative middle piece cancels one positive piece. A sketch of v against t shows you where to split.