

Linear combinations of random variables

A-Level Maths 9709 · Topic 6.2 revision notes

Everything from the 1 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Linear combinations of random variables

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1 Linear combinations of random variables

Syllabus 6.2

$E(aX + bY) = aE(X) + bE(Y)$ always. For independent X and Y , $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$, so variances ADD even for a difference. Constants shift the mean but not the variance. Combinations of independent normals are normal; sums of independent Poissons are Poisson.

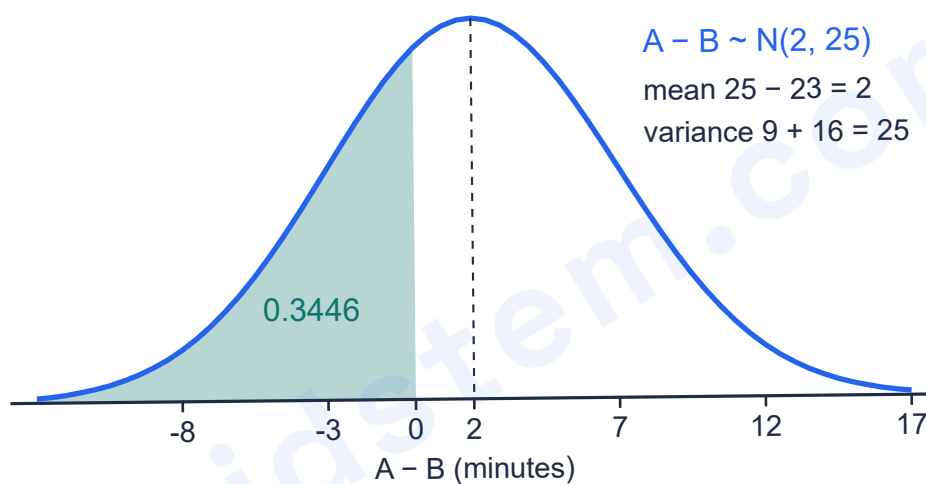


Fig. 1: Ann's time $A \sim N(25, 9)$ and Ben's time $B \sim N(23, 16)$. The difference $A - B \sim N(2, 25)$; the shaded area $P(A - B < 0) = 0.3446$ is the probability that Ann is faster

Worked example: Mean and variance of a transformed variable

$E(X) = 12$ and $\text{Var}(X) = 4$. Find the mean and variance of $Y = 3X - 5$.

1. $E(Y) = 3 \times 12 - 5 = 31$.
2. $\text{Var}(Y) = 3^2 \times 4 = 36$. The -5 has no effect on the spread.

Answer: $E(Y) = 31$, $\text{Var}(Y) = 36$

Write the combination, then its mean and variance.

- ✓ **Define** the combination, e.g. $D = X_1 + X_2 + X_3 - 2Y$, so the question becomes $P(D > 0)$.
- ✓ **Mean:** apply the coefficients directly.
- ✓ **Variance:** square every coefficient and ADD, for sums and differences alike.
- ✓ **Separate items or one item scaled?** n items: $n\sigma^2$; one item times n : $n^2\sigma^2$.
- ✓ **Shape:** normal stays normal; Poisson sums stay Poisson.
- ✓ **Standardise** with the square root of the variance.

Watch out!

$\text{Var}(3X - 5)$ is not $3 \times 4 - 5$, and not 3×4 . The factor is squared and the constant disappears: $9 \times 4 = 36$.

The commonest Paper 6 error: $\text{Var}(A - B) = 9 - 16$. A variance can never be negative; for independent variables they always add.

Before adding Poisson parameters, put them on the same interval. A rate of 2 per hour and 0.5 per 10 minutes give $\text{Po}(2 + 3) = \text{Po}(5)$ for an hour, not $\text{Po}(2.5)$.