

Newton's laws of motion

A-Level Maths 9709 · Topic 4.4 revision notes

Everything from the 3 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Newton's laws of motion

2. Motion on an inclined plane

3. Connected particles and pulleys

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1 Newton's laws of motion

Syllabus 4.4

Newton's second law: resultant force $= ma$, resolved in the direction of the acceleration. Perpendicular to the motion the forces still balance. Weight is $W = mg$ with $g = 10 \text{ m s}^{-2}$, and a sliding particle has friction $F = \mu R$.

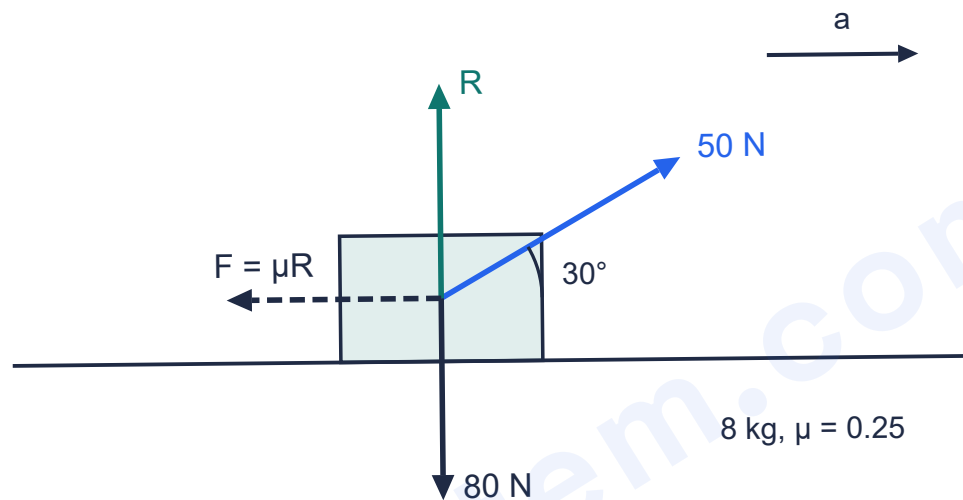


Fig. 1: an 8 kg block pulled along rough horizontal ground by a 50 N force at 30° to the horizontal (angle to scale). The pull reduces R to 55 N, so friction is 13.75 N

Worked example: A driving force against resistance

A particle of mass 5 kg on a smooth horizontal surface is pulled by a horizontal force of 20 N against a resistance of 5 N. Find its acceleration.

1. Resultant force in the direction of motion $= 20 - 5 = 15 \text{ N}$.
2. $15 = 5a$.

Answer: $a = 3 \text{ m s}^{-2}$

Diagram, resolve, $F = ma$, suvat.

- ✓ **Diagram:** all forces, plus an arrow for the acceleration.
- ✓ **Perpendicular to motion:** forces balance; this usually gives R .
- ✓ **Friction:** moving means $F = \mu R$, opposing the motion.
- ✓ **Along the motion:** resultant force = ma .
- ✓ **Motion:** use suvat with the acceleration you found (or the reverse).

Watch out!

$F = ma$ uses the **RESULTANT** force. Writing $50 = 8a$, or $50 \cos 30^\circ = 8a$, ignores friction and is the most common lost mark.

Keep signs consistent. If the particle is slowing down, a is negative in your positive direction, and so is the resultant force.

"Moving upwards" does not mean "accelerating upwards". A lift moving up while slowing down has a **DOWNWARD** acceleration, so the reaction is less than the weight.

2 Motion on an inclined plane

Syllabus 4.4

Along the plane: resultant force $= ma$. Perpendicular: $R = mg \cos \alpha$ (plus or minus any other perpendicular components). Moving up a rough plane, friction acts down it, so $mg \sin \alpha + \mu R$ slows the particle; moving down, friction acts up, so the driving force is $mg \sin \alpha - \mu R$.

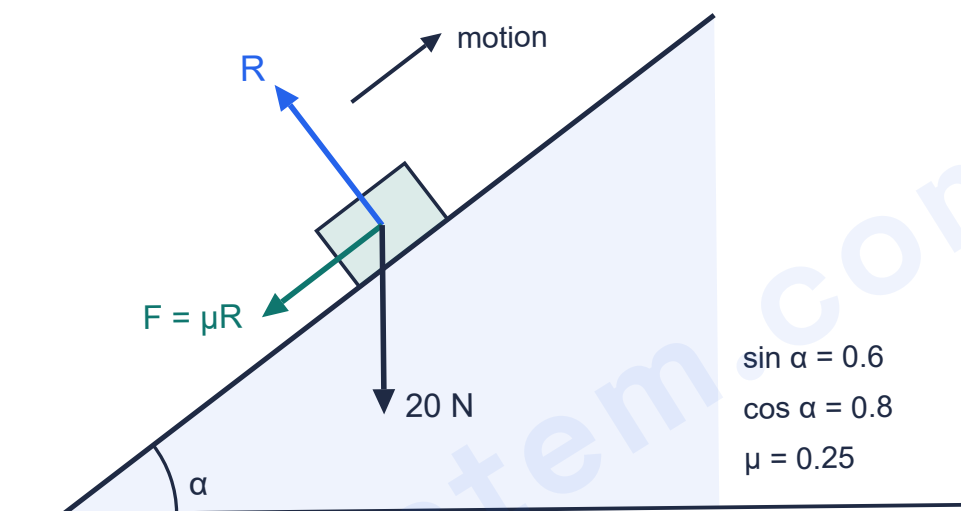


Fig. 1: a 2 kg particle moving UP a rough plane with $\sin \alpha = 0.6$ ($\alpha = 36.9^\circ$, drawn to scale). Friction acts down the plane, against the motion

Worked example: Released on a smooth plane

A particle is released from rest on a smooth plane inclined at 30° to the horizontal. Find its speed after it has slid 4 m and the time taken.

1. Down the plane: $mg \sin 30^\circ = ma$, so $a = 10 \times 0.5 = 5 \text{ m s}^{-2}$.
2. $v^2 = 0 + 2(5)(4) = 40$, so $v = 6.32 \text{ m s}^{-1}$.
3. $t = \frac{v}{a} = \frac{6.325}{5} = 1.26 \text{ s}$.

Answer: 6.32 m s^{-1} after 1.26 s.

Perpendicular for R , along for $F = ma$.

- ✓ **Weight:** $mg \sin \alpha$ down the plane, $mg \cos \alpha$ into it.
- ✓ **Perpendicular:** $R = mg \cos \alpha \pm$ components of other forces.
- ✓ **Friction:** μR , opposite to the motion (it switches direction when the particle turns round).
- ✓ **Along the plane:** resultant $= ma$, then suvat.
- ✓ **At rest at the top?** It stays if $\mu \geq \tan \alpha$.

Watch out!

$a = g \sin \alpha$, not $g \cos \alpha$. Check with a limiting case: on a vertical "slope" ($\alpha = 90^\circ$) the particle falls freely with $a = g$, and $\sin 90^\circ = 1$.

After a particle stops at the top, do not assume it slides back. Compare the weight component $mg \sin \alpha$ with the maximum friction $\mu mg \cos \alpha$: it stays put if $\mu \geq \tan \alpha$.

Using $R = mg \cos \alpha$ when the applied force has a component perpendicular to the plane. In the second example that gives friction 5.20 N instead of 9.20 N.

3 Connected particles and pulleys

Syllabus 4.4

A light inextensible string has the same tension all along it (also over a smooth pulley), and the particles it joins have accelerations of equal size. Write $F = ma$ for EACH particle in its own direction of motion, then solve the equations together. A tow-bar can be in tension or in thrust.

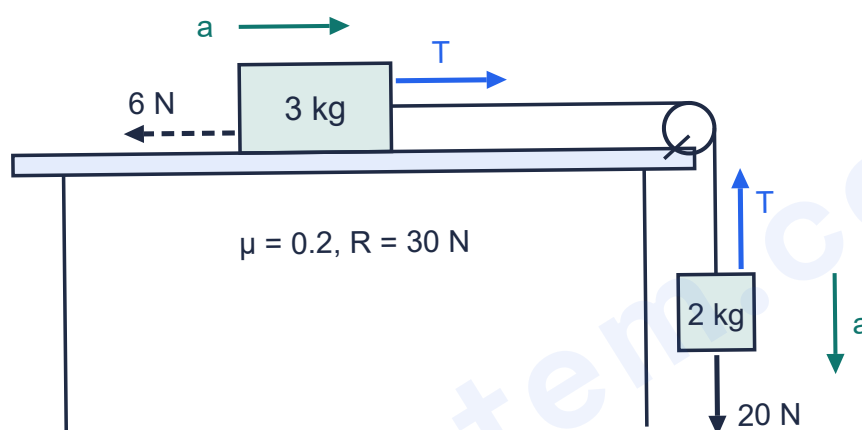


Fig. 1: a 3 kg block on a rough table ($\mu = 0.2$) joined by a light inextensible string over a smooth pulley to a hanging 2 kg block. The tension T is the same on both sides; both blocks have the same acceleration a

Worked example: Tension in a tow-rope

A car of mass 1200 kg tows a trailer of mass 400 kg along a straight horizontal road with a light rope. The driving force is 2500 N, and the resistances are 300 N on the car and 200 N on the trailer. Find the acceleration and the tension.

1. Whole system: $2500 - 300 - 200 = 1600a$, so $a = 1.25 \text{ m s}^{-2}$.
2. Trailer alone: $T - 200 = 400(1.25)$.
3. $T = 700 \text{ N}$.
4. (Check with the car: $2500 - 300 - 700 = 1500 = 1200 \times 1.25$.)

Answer: $a = 1.25 \text{ m s}^{-2}$, $T = 700 \text{ N}$.

$F = ma$ for each particle, then add.

- ✓ **Diagram:** forces on each particle separately, with T on both ends of the string.
- ✓ **Directions:** each particle's own direction of motion is positive.
- ✓ **Equations:** $F = ma$ for each particle (or the whole system plus one particle).
- ✓ **Solve:** add to eliminate T , find a , then T .
- ✓ **Slack string:** $T = 0$; continue with the velocity at that instant.

Watch out!

A rope can only pull. A tow-bar can push, and when the car brakes hard it does. If your tension comes out negative, the connector is in thrust (for a rope, that would mean it goes slack).

The tension is NOT the weight of the hanging particle. If it were, B would have no resultant force and could not accelerate. In Fig. 1, $T = 14.4 \text{ N}$, less than B's 20 N weight.

After the string goes slack, the rising particle does NOT stop at once; it keeps moving up and decelerates at g . Forgetting this second stage gives 1.2 m instead of 1.5 m .