

The normal distribution

A-Level Maths 9709 · Topic 5.5 revision notes

Everything from the 2 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. The normal distribution

2. The normal approximation to the binomial

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1 The normal distribution

Syllabus 5.5

If $X \sim N(\mu, \sigma^2)$, then $Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$ and $P(X < x) = \Phi\left(\frac{x - \mu}{\sigma}\right)$. Use symmetry, $\Phi(-z) = 1 - \Phi(z)$, for negative z , and always sketch the curve.

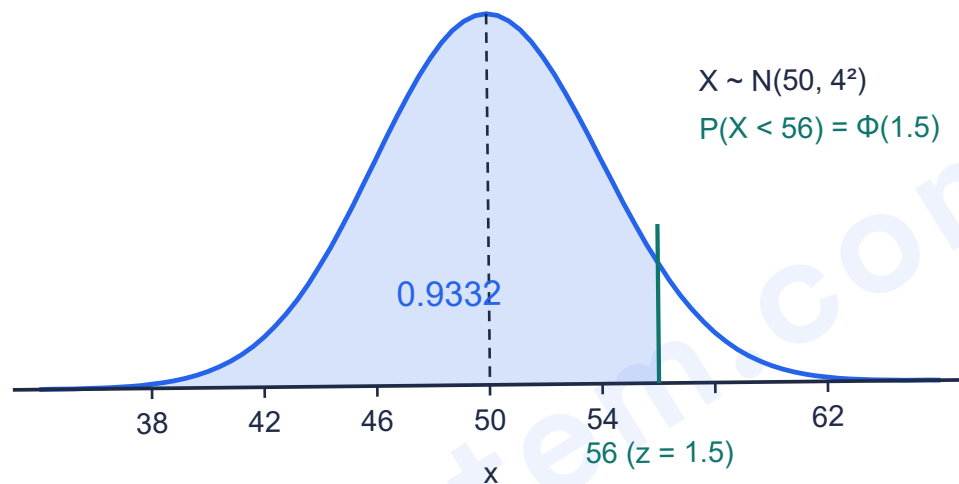


Fig. 1: $X \sim N(50, 4^2)$ drawn to scale. The value 56 is 1.5 standard deviations above the mean, and the shaded area to its left is $\Phi(1.5) = 0.9332$

Worked example: Less than

$X \sim N(50, 4^2)$. Find $P(X < 56)$.

1. $z = \frac{56 - 50}{4} = 1.5$.

2. $P(X < 56) = \Phi(1.5)$, read from the table (Fig. 1).

Answer: 0.9332

Sketch first, then $z = \frac{x - \mu}{\sigma}$.

- ✓ **Sketch:** mark μ , the value(s) and the shaded area.
- ✓ **Standardise:** divide by σ , not σ^2 ; z to 3 d.p.
- ✓ **Look up:** $\Phi(z)$ is the area to the left; use $1 - \Phi$ and symmetry for others.
- ✓ **Backwards:** probability $\rightarrow z$ from the table, with the sign from the sketch.
- ✓ **Two unknowns:** two equations $x = \mu + z\sigma$, solved together.

Watch out!

Dividing by the variance.

$N(50, 16)$ means $\sigma^2 = 16$, so

$\sigma = 4$; $\frac{56 - 50}{16}$ is wrong.

Writing $\Phi(0.75) - \Phi(1)$ for $P(46 < X < 53)$. The left end has $z = -1$, not $+1$; a sketch shows the area must be bigger than $\Phi(0.75) - 0.5$.

Using $z = 0.1$ or $z = 0.8$ in the equation. Those are probabilities; you must convert them to z -values with the table first.

Giving both z -values the same sign. A value below the mean always has a negative z ; a quick sketch with 20 to the left of μ and 40 to the right prevents this.

2 The normal approximation to the binomial

Syllabus 5.5

If $X \sim B(n, p)$ with $np > 5$ and $nq > 5$ (where $q = 1 - p$), then approximately $X \sim N(np, npq)$. Each whole number r is represented by the interval from $r - 0.5$ to $r + 0.5$: this is the continuity correction.

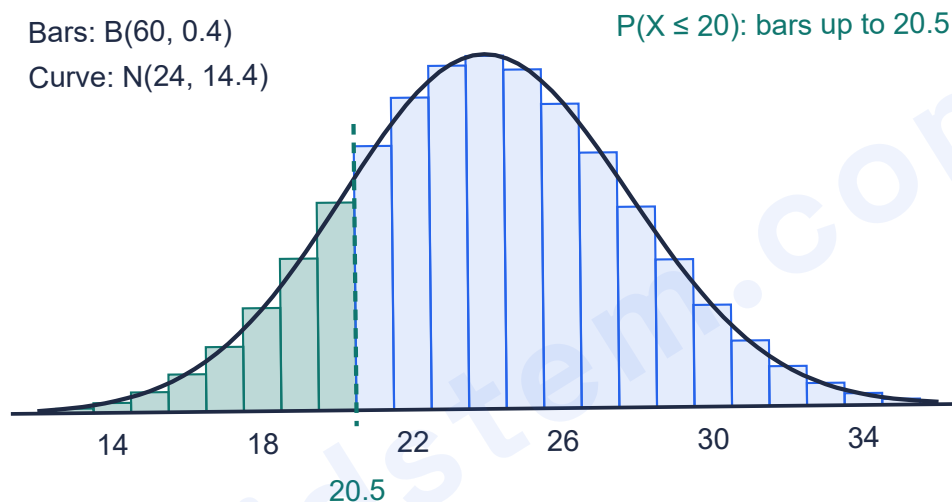


Fig. 1: $B(60, 0.4)$ with the normal curve $N(24, 14.4)$ drawn to scale. The bar for 20 runs from 19.5 to 20.5, so ' $X \leq 20$ ' means all the area to the left of 20.5

Worked example: Setting up the approximation

$X \sim B(60, 0.4)$. Show that a normal approximation is suitable and state its distribution.

1. $np = 60 \times 0.4 = 24 > 5$ and $nq = 60 \times 0.6 = 36 > 5$.
2. Mean $np = 24$; variance $npq = 60 \times 0.4 \times 0.6 = 14.4$.

Answer: $X \approx Y \sim N(24, 14.4)$, with $\sigma = \sqrt{14.4} = 3.795$

Check, approximate, correct, standardise.

- ✓ **Define:** $X \sim B(n, p)$ in context.
- ✓ **Check:** $np > 5$ and $nq > 5$.
- ✓ **Approximate:** $Y \sim N(np, npq)$; $\sigma = \sqrt{npq}$.
- ✓ **Correct:** each whole number r covers $r - 0.5$ to $r + 0.5$; decide whether each boundary is included.
- ✓ **Standardise:** $z = \frac{y - np}{\sqrt{npq}}$, then use Φ and symmetry.

Watch out!

Using $\sqrt{np}$ or npq as the standard deviation. The variance is $npq = 14.4$, so $\sigma = \sqrt{14.4} = 3.795$.

Correcting in the wrong direction. Ask 'is the boundary value included?' For $X \geq 30$ it is, so you go DOWN to 29.5 to take in its whole bar.

Answering 0 for $P(X = 25)$. That is true for a continuous variable, but X is binomial; the continuity correction turns 25 into the interval 24.5 to 25.5.

Treating 'more than 25' as $P(Y > 24.5)$. 25 itself is not included, so the boundary moves up to 25.5.