

Probability

A-Level Maths 9709 · Topic 5.3 revision notes

Everything from the 2 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

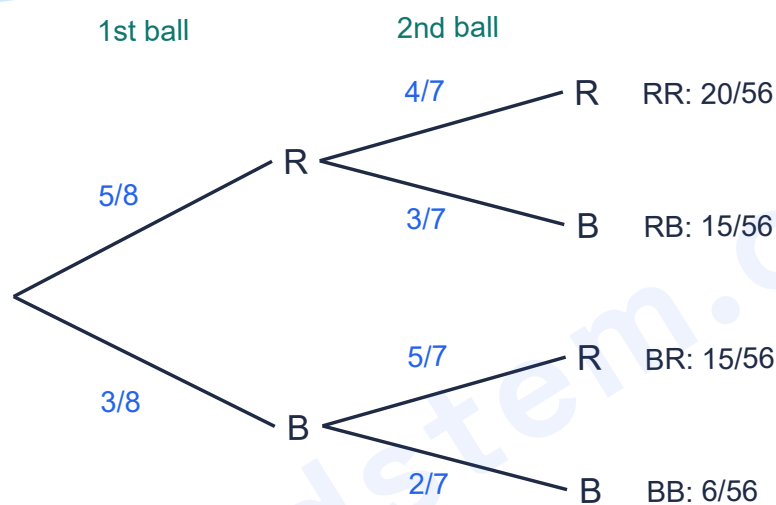
1. Probability: sample spaces, exclusive events
2. Conditional probability and independence and tree diagrams

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1 Probability: sample spaces, exclusive events and tree diagrams

Syllabus 5.3

For equally likely outcomes, $P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$. If A and B are exclusive, $P(A \cup B) = P(A) + P(B)$. Along a tree, multiply; across separate routes, add. $P(\text{at least one}) = 1 - P(\text{none})$.



Same colour: $20/56 + 6/56 = 26/56 = 13/28$

Fig. 1: two balls taken without replacement from a bag of 5 red and 3 blue. The second-stage probabilities change because one ball has gone; the four end results add up to 1

Worked example: Totals of two dice

Two fair dice are thrown. Find the probability that (a) the total is 8, (b) the total is at least 10.

1. The sample space has $6 \times 6 = 36$ equally likely outcomes.
2. (a) Total 8: (2,6), (3,5), (4,4), (5,3), (6,2), which is 5 outcomes.
3. (b) Total 10: 3 outcomes; 11: 2; 12: 1. That is 6 outcomes.

Answer: (a) $\frac{5}{36}$ (b) $\frac{6}{36} = \frac{1}{6}$

Count, then add or multiply.

- ✓ **Equally likely:** favourable outcomes $\div$ total outcomes.
- ✓ **Or, exclusive:** add. **And, independent:** multiply.
- ✓ **Trees:** multiply along a route, add the routes; check later branches for 'without replacement'.
- ✓ **At least one:** $1 - P(\text{none})$.
- ✓ **Several picked at once:** $\frac{\text{favourable selections}}{\text{total selections}}$ using ${}^nC_r$.

Watch out!

Saying $P(\text{total } 8) = \frac{1}{11}$ because there are 11 possible totals. The totals 2 to 12 are NOT equally likely; only the 36 ordered pairs are.

Adding probabilities of events that are NOT exclusive. With $P(A) = 0.6$ and $P(B) = 0.7$, 'adding' gives 1.3, which is impossible; the overlap has been counted twice.

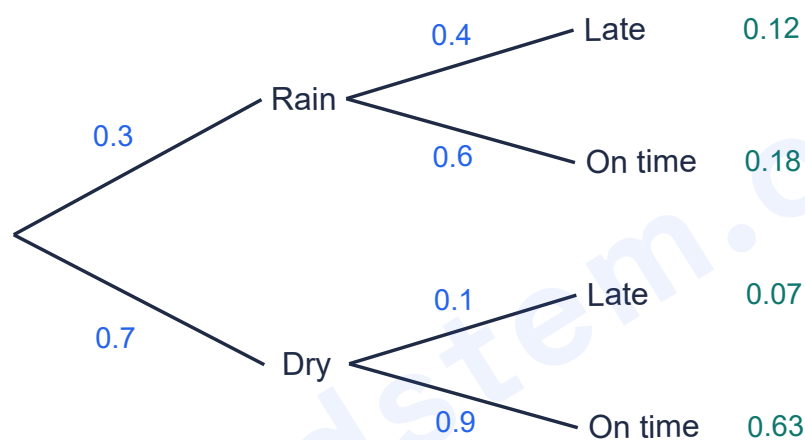
Keeping the same denominators on the second branches. Without replacement there are only 7 balls left, so the second-stage fractions are sevenths.

Forgetting the different orders when using a tree for 'exactly two red'. RRB alone gives $\frac{5}{28}$; there are 3 orders (RRB, RBR, BRR). Combinations count them automatically.

2 Conditional probability and independence

Syllabus 5.3

$P(A | B) = \frac{P(A \cap B)}{P(B)}$: the probability of A once you know B has happened. Events are independent exactly when $P(A \cap B) = P(A) \times P(B)$, which is the same as $P(A | B) = P(A)$.



$$P(\text{Rain} | \text{Late}) = 0.12 \div (0.12 + 0.07) = 0.632$$

Fig. 1: the tree is drawn forwards (weather, then lateness). Given that the bus is late, only the two 'Late' routes remain, and the rain route is 0.12 of their 0.19

Worked example: From a two-way table

The table shows 120 students and whether they study French. One student is chosen at random. Find (a) $P(\text{studies French} | \text{female})$, (b) $P(\text{female} | \text{studies French})$.

- (a) 'Given female' restricts us to the 60 females, of whom 30 study French: $\frac{30}{60}$.
- (b) 'Given French' restricts us to the 48 French students, of whom 30 are female: $\frac{30}{48}$.
- With the formula: $P(F | \text{French}) = \frac{30/120}{48/120} = \frac{30}{48}$. Same answer.

Answer: (a) $\frac{1}{2}$ (b) $\frac{5}{8}$

Route wanted ÷ all routes allowed.

- ✓ **Formula:** $P(A | B) = \frac{P(A \cap B)}{P(B)}$.
- ✓ **Tables:** the condition picks the row or column total to divide by.
- ✓ **Trees:** find $P(B)$ by adding every route through B , then divide.
- ✓ **Independence:** calculate $P(A \cap B)$ and $P(A)P(B)$; state equal or not.
- ✓ **Exclusive:** $P(A \cap B) = 0$, so (with non-zero probabilities) not independent.

Watch out!

Mixing up $P(A | B)$ and $P(B | A)$. They have the same numerator but different denominators: $\frac{1}{2}$ and $\frac{5}{8}$ above.

Answering 0.4. That is $P(\text{late} | \text{rain})$, read straight off the branch; the question asks for $P(\text{rain} | \text{late})$, which needs the whole calculation.

Deciding independence by intuition ('the dice are separate, so the events must be independent').

Events defined on the same throw can go either way: A is independent of B but not of C .

Writing 'independent, because they cannot happen together'. That describes exclusive events, and it shows the events are NOT independent.