

Coordinate geometry

IGCSE Maths 0580 · Topic 3 revision notes

Everything from the 6 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

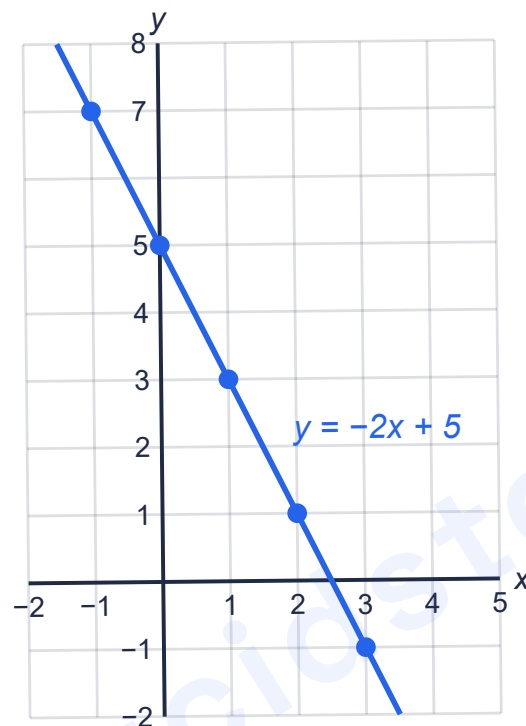
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|---|---|
| 1. <u>Coordinates and drawing linear graphs</u> | 4. <u>The equation of a straight line</u> |
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1 Coordinates and drawing linear graphs

Syllabus 3.2

A point is written (x, y) : across first, then up or down. To draw a linear graph, choose some x values, work out y for each, plot the points and join them with ONE ruled straight line. If a point is off the line, one of your calculations is wrong.



x	-1	0	1	2	3
y	7	5	3	1	-1

plot each (x, y) pair,
then join with a ruler

Fig. 1: the table of values for $y = -2x + 5$ gives five points, and all five lie on one straight line

Worked example: Finding the fourth corner

$A(-2, 3)$, $B(4, 3)$ and $C(4, -1)$ are three corners of a rectangle $ABCD$. Find the coordinates of D .

1. A and B have the same y value, so AB is horizontal. B and C have the same x value, so BC is vertical.
2. D must be directly below A , so it has the same x value as A : $x = -2$.
3. D must be level with C , so it has the same y value as C : $y = -1$.

Answer: $D(-2, -1)$

Choose x , work out y , plot, rule.

- ✓ **Coordinates:** (x, y) , across first, then up or down.
- ✓ **Table:** substitute each x carefully; brackets round negative numbers.
- ✓ **Pattern check:** the y values change by the same amount each step.
- ✓ **Plot and rule:** one straight line through every point, across the full range.
- ✓ **Special lines:** $x = k$ is vertical, $y = k$ is horizontal.

Watch out!

Writing the coordinates the wrong way round. (x, y) is always across first, then up: 'along the corridor, then up the stairs'. $(-1, 3)$ and $(3, -1)$ are in different quadrants.

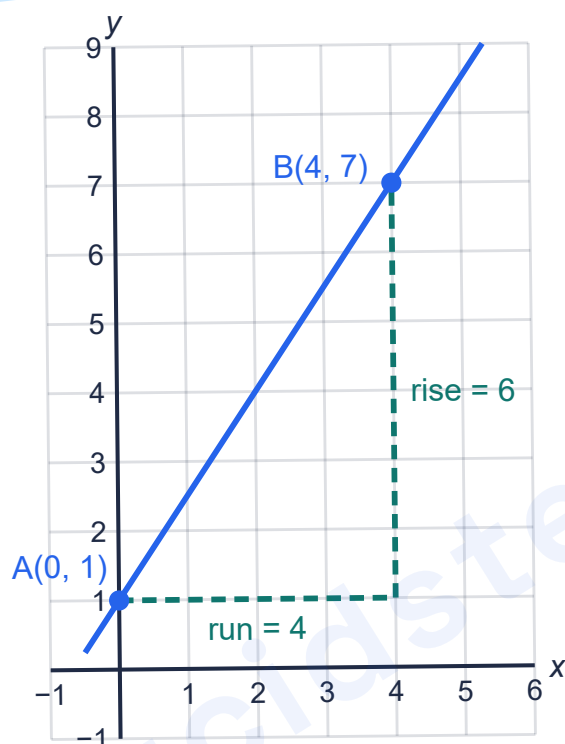
Sign slips with negative x : $-2 \times (-1)$ is $+2$, not -2 . If one value breaks the steady pattern in your table, recheck that column before you plot.

Mixing up $x = 3$ and $y = 3$. The line $x = 3$ is VERTICAL (every point has $x = 3$), even though it crosses the x -axis. Picture a few points on it before you draw.

2 Gradient of a straight line

Syllabus 3.3

Gradient = $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$. Uphill from left to right is positive, downhill is negative, a horizontal line has gradient 0, and a vertical line has no gradient (it is undefined).



$$\begin{aligned}\text{gradient} &= \text{rise} \div \text{run} \\ &= 6 \div 4 \\ &= 1.5\end{aligned}$$

Fig. 1: from $A(0, 1)$ to $B(4, 7)$ the line rises 6 for a run of 4, so its gradient is $6 / 4 = 1.5$

Worked example: Uphill line from a grid

A straight line passes through the grid points $(0, 1)$ and $(4, 7)$. Find its gradient.

1. Run: from $x = 0$ to $x = 4$ is 4 units across.

2. Rise: from $y = 1$ to $y = 7$ is 6 units up.

3. Gradient = $\frac{6}{4} = \frac{3}{2}$.

Answer: $\frac{3}{2}$ or 1.5 (Fig. 1)

Change in y over change in x

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- ✓ **Grid:** draw a right-angled triangle under the line between two exact grid points.
- ✓ **Coordinates (Extended):** $m = \frac{y_2 - y_1}{x_2 - x_1}$, same order top and bottom.
- ✓ **Sign:** uphill positive, downhill negative.
- ✓ **Special lines:** horizontal 0, vertical undefined.
- ✓ **Meaning:** the gradient is the change in y for each 1 unit of x , with units y -units per x -unit.

Watch out!

Counting squares when the axes have different scales. If one square across is 2 units and one square up is 10 units, read the actual values off the axes, not the number of squares.

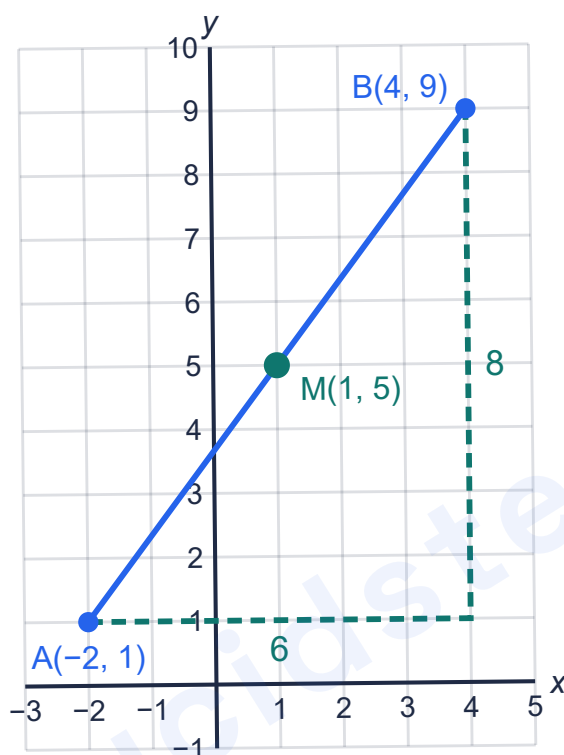
Taking the y values in one order and the x values in the other order. If you start with y_2 on top, you must start with x_2 underneath, or the sign flips. And it is y over x , never x over y .

A horizontal line has gradient 0 (no rise at all), but a VERTICAL line does not have gradient 0: its run is 0, and you cannot divide by 0, so its gradient is undefined.

3 Length and midpoint of a line segment

Syllabus 3.4

For $A(x_1, y_1)$ and $B(x_2, y_2)$: length $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Length uses differences; midpoint uses sums.



$$\begin{aligned}\text{length } AB \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{100} = 10\end{aligned}$$

$$\begin{aligned}\text{midpoint } M \\ x: (-2 + 4) \div 2 &= 1 \\ y: (1 + 9) \div 2 &= 5 \\ M &= (1, 5)\end{aligned}$$

Fig. 1: from $A(-2, 1)$ to $B(4, 9)$ is 6 across and 8 up, so $AB = 10$; the midpoint $M(1, 5)$ averages the coordinates

Worked example: A whole-number length

Find the length of the line segment joining $A(-2, 1)$ and $B(4, 9)$.

1. Change in x : $4 - (-2) = 6$.
2. Change in y : $9 - 1 = 8$.
3. $AB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}$.

Answer: $AB = 10$ units (Fig. 1)

Differences for length, sums for midpoint.

- ✓ **Steps:** work out the change in x and the change in y , with brackets for negatives.
- ✓ **Length:**
$$\sqrt{(\text{change in } x)^2 + (\text{change in } y)^2}$$
- ✓ **Midpoint:** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.
- ✓ **Missing end point:** repeat the step from the known end to the midpoint.
- ✓ **Accuracy:** exact surd or 3 significant figures.

Watch out!

Adding before squaring. $\sqrt{6^2 + 8^2}$ is 10, but $6 + 8 = 14$ and $\sqrt{6 + 8} = 3.74$ are both wrong. Square each step first, add, then take the square root of the total.

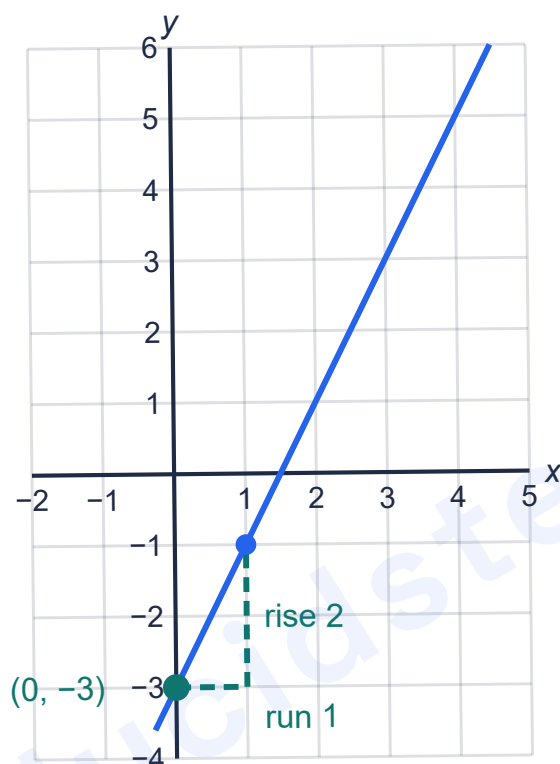
Halving the difference instead of the sum. For $(-3, 8)$ and $(9, -2)$, $\frac{9 - (-3)}{2} = 6$ is half the run, not the midpoint's x -coordinate. The midpoint is an average: ADD, then halve.

Forgetting the negative square root. $(k - 2)^2 = 25$ gives $k - 2 = \pm 5$: a point 13 units away can be on either side, so there are usually two answers.

4 The equation of a straight line

Syllabus 3.5

In $y = mx + c$, m is the gradient and c is the y -intercept (the line crosses the y -axis at $(0, c)$). To find an equation: get the gradient, then substitute one point to find c . A vertical line has equation $x = k$.



$$y = 2x - 3$$

$$\begin{aligned}\text{gradient } m \\ &= 2 \div 1 = 2\end{aligned}$$

$$\begin{aligned}\text{y-intercept } c \\ &= -3\end{aligned}$$

Fig. 1: $y = 2x - 3$ crosses the y -axis at $(0, -3)$ and rises 2 for every 1 across, so $m = 2$ and $c = -3$

Worked example: Reading the gradient and intercept

Write down the gradient and y -intercept of (a) $y = 6x + 3$ (b) $y = 5 - 2x$.

- (a) The coefficient of x is 6 and the constant is 3.
- (b) Rewrite as $y = -2x + 5$: the coefficient of x is -2 (the minus sign belongs to it).

Answer: (a) $m = 6$, $c = 3$ (b) $m = -2$, $c = 5$

Gradient first, then substitute a point.

- ✓ **Read:** in $y = mx + c$, m is the gradient and c the y -intercept.
- ✓ **Rearrange (Extended):** make y the subject, dividing every term.
- ✓ **Gradient:** from the graph, or $\frac{y_2 - y_1}{x_2 - x_1}$ from two points.
- ✓ **Intercept:** substitute one point into $y = mx + c$ and solve for c .
- ✓ **Check:** the other point must fit; give the equation fully simplified.

Watch out!

Taking the first number as the gradient. In $y = 5 - 2x$ the gradient is -2 , not 5 ; the gradient is always the number in front of x , with its sign.

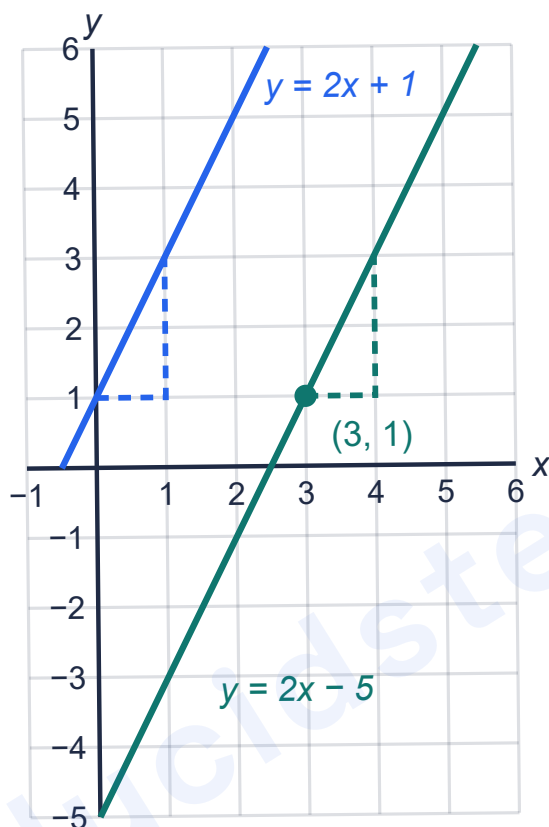
Dividing only some of the terms. From $4y = -5x + 8$, the equation $y = -5x + 2$ divides only the 8 by 4 . Every term on the right must be divided.

Putting the coordinates in the wrong places: for the point $(2, 1)$, $x = 2$ and $y = 1$. Writing $2 = 3 \times 1 + c$ gives the wrong c . Always check your final equation with the point.

5 Parallel lines

Syllabus 3.6

Parallel lines have equal gradients. To find the line parallel to $y = mx + c$ through a point: keep the same m , substitute the point to find the new c .



same gradient: 2
(rise 2 for
every 1 across)

different
y-intercepts:
1 and -5

Fig. 1: $y = 2x + 1$ and $y = 2x - 5$ both rise 2 for every 1 across, so they are parallel; the second one passes through $(3, 1)$

Worked example: Spotting parallel lines

Which of these lines are parallel? A: $y = 3x + 2$ B: $y = 2x + 3$ C: $y = 3x - 5$ D: $y = 5 - 3x$

1. Gradients: A is 3, B is 2, C is 3.
2. D is $y = -3x + 5$, so its gradient is -3 , not 3.
3. Only A and C have the same gradient.

Answer: A and C

Same m , new C .

- ✓ **Gradient:** write the given line as $y = mx + c$ and read m .
- ✓ **Parallel:** the new line has the same gradient m .
- ✓ **Intercept:** substitute the given point into $y = mx + c$ to find the new c .
- ✓ **Shortcut (Extended):** parallel to $ax + by = c$ is $ax + by = k$; put the point in to find k .
- ✓ **Proof:** equal gradients show that two sides are parallel.

Watch out!

Using the point's y -coordinate as the intercept. The line through $(1, -3)$ is NOT $y = 4x - 3$: the point is not on the y -axis, so you must substitute to find c .

Reading the coefficient of x in $2x + 3y = 6$ as the gradient.

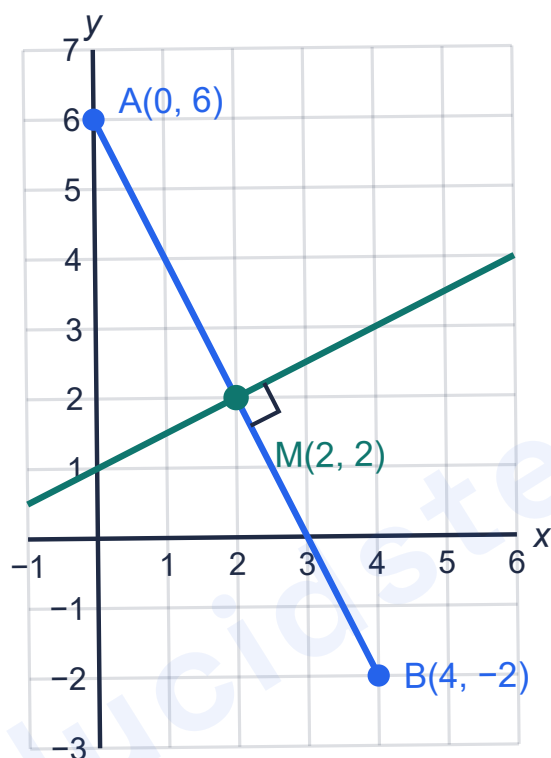
The gradient is $-\frac{2}{3}$, not 2:
rearrange to $y = \dots$ before you read it.

Dividing by a negative and losing the sign. From $-2y = -6x + 5$, every term is divided by -2 :
 $y = 3x - 2.5$. Check the sign of your gradient by picturing which way the line slopes.

6 Perpendicular lines and bisectors

Syllabus 3.7

If two lines are perpendicular, $m_1 \times m_2 = -1$, so the perpendicular gradient is $-\frac{1}{m}$: flip the fraction and change the sign. The perpendicular bisector of AB passes through the midpoint of AB with the perpendicular gradient.



AB: gradient -2

bisector:

gradient $1/2$

$$(-2 \times 1/2 = -1)$$

$$y = x/2 + 1$$

Fig. 1: AB has gradient -2 ; its perpendicular bisector passes through the midpoint $M(2, 2)$ with gradient $1/2$, giving $y = x/2 + 1$

Worked example: Perpendicular gradients

Find the gradient of a line perpendicular to (a) $y = 3x - 2$ (b) $2y = 3x + 1$.

1. (a) The gradient is 3. Flip: $\frac{1}{3}$. Change sign: $-\frac{1}{3}$.
2. (b) Make y the subject: $y = \frac{3}{2}x + \frac{1}{2}$, so the gradient is $\frac{3}{2}$.
3. Flip and change sign: $-\frac{2}{3}$. Check: $\frac{3}{2} \times \left(-\frac{2}{3}\right) = -1$.

Answer: (a) $-\frac{1}{3}$ (b) $-\frac{2}{3}$

Flip, change the sign, substitute.

- ✓ **Gradient:** write the given line as $y = mx + c$ and read m .
- ✓ **Perpendicular:** new gradient $-\frac{1}{m}$; check the product is -1 .
- ✓ **Through a point:** substitute it into $y = mx + c$ to find c .
- ✓ **Bisector of AB :** midpoint of AB plus the gradient perpendicular to AB .
- ✓ **Tidy:** clear fractions if asked for the form $ax + by = c$.

Watch out!

Doing only half the recipe. The perpendicular to gradient 3 is not -3 (sign only) and not $\frac{1}{3}$ (flip only). It is $-\frac{1}{3}$. Multiply your two gradients: you must get -1 .

Using the original gradient by mistake when you substitute. Write the NEW gradient down on its own line before finding c , so you cannot slip back to the old one.

Putting A or B into the equation instead of the midpoint. The bisector passes through M , not through the end points, so substitute the midpoint to find c .