

# Geometry

## IGCSE Maths 0580 · Topic 4 revision notes

Everything from the 10 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

- |                                                      |                                                                        |
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# 1 Geometrical terms: shapes, solids and nets

## Syllabus 4.1

Name a shape by the properties it **MUST** have, not by how it looks: a trapezium has exactly one pair of parallel sides, a rhombus has four equal sides, and a square is a special rectangle **AND** a special rhombus.

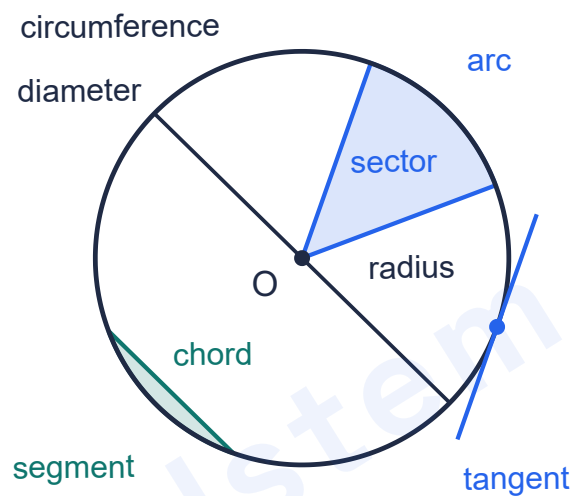


Fig. 1: the parts of a circle with centre O. A sector is cut off by two radii; a segment is cut off by a chord

### Worked example: Naming angles

Classify the angles  $35^\circ$ ,  $90^\circ$ ,  $128^\circ$  and  $215^\circ$ . Then find the reflex angle that goes with  $128^\circ$ .

1.  $35^\circ$  is between  $0^\circ$  and  $90^\circ$ , so it is acute.
2.  $90^\circ$  is a right angle.
3.  $128^\circ$  is between  $90^\circ$  and  $180^\circ$ , so it is obtuse.
4.  $215^\circ$  is between  $180^\circ$  and  $360^\circ$ , so it is reflex.
5. The two angles between the same arms make a full turn, so the reflex partner of  $128^\circ$  is  $360^\circ - 128^\circ = 232^\circ$ .

**Answer:** acute, right, obtuse, reflex; reflex partner  $232^\circ$

## Name it by what it must have.

- ✓ **Angles:** acute below  $90^\circ$ , obtuse  $90^\circ$  to  $180^\circ$ , reflex  $180^\circ$  to  $360^\circ$ .
- ✓ **Quadrilaterals:** check parallel sides, equal sides, right angles and diagonals in turn.
- ✓ **Solids:** count faces, then edges, then vertices; check  $F + V - E = 2$ .
- ✓ **Circles:** sector = two radii and an arc; segment = chord and arc.
- ✓ **Use the exact word:** the mark is for the correct term, spelt clearly.

## Watch out!

A reflex angle is NOT just a big obtuse angle.  $170^\circ$  is obtuse; anything over  $180^\circ$  is reflex. When asked for a reflex angle, subtract the inside angle from  $360^\circ$ , not from  $180^\circ$ .

A kite's equal sides are next to each other (adjacent); a parallelogram's equal sides are opposite. And a trapezium has exactly ONE pair of parallel sides: a parallelogram is not a trapezium in IGCSE.

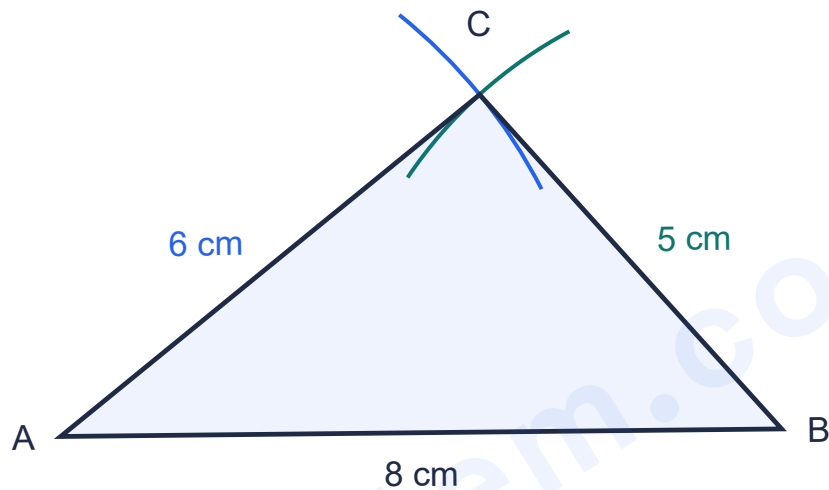
Do not count the same edge twice. Every edge is shared by exactly two faces, so counting the edges of each face and adding gives double the true number.

Sector and segment are the classic mix-up. A sector always reaches the centre (two radii); a segment never does unless the chord is a diameter.

## 2 Ruler and compasses constructions

### Syllabus 4.2

To construct a triangle from three sides: draw one side with a ruler, then draw an arc from each end with the compasses set to the other two lengths. The arcs cross at the third vertex. Leave the arcs showing.



arc: centre A, radius 6 cm

arc: centre B, radius 5 cm

Fig. 1: triangle ABC with  $AB = 8$  cm,  $AC = 6$  cm and  $BC = 5$  cm, drawn to scale. The two construction arcs cross at C

### Worked example: Drawing a reflex angle

Explain how to draw an angle of  $250^\circ$  with a  $180^\circ$  protractor.

1. The reflex angle and the angle on the other side make a full turn:  $360^\circ - 250^\circ = 110^\circ$ .
2. Draw a line, and at one end measure and draw an angle of  $110^\circ$ .
3. The reflex angle is the angle OUTSIDE this  $110^\circ$  angle; mark it with an arc and label it  $250^\circ$ .

**Answer:** Draw  $110^\circ$  and mark the outside angle as  $250^\circ$ .

### *Line, arc, arc, join.*

- ✓ **Check first:** the two shorter sides must add to more than the longest.
- ✓ **Base:** draw one side accurately with a ruler.
- ✓ **Arcs:** compasses set to each other length, centred on the matching end.
- ✓ **Join and keep:** join to the crossing point; leave the arcs showing.
- ✓ **Nets:** surface area = total area of every face in the net.

### *Watch out!*

Reading the wrong scale on the protractor is the most common slip:  $50^\circ$  read as  $130^\circ$ . Always decide acute or obtuse by eye before you read the number.

Rubbing out the construction arcs loses marks even when the triangle is perfect. Cambridge says construction arcs must be shown. Also, set each arc's radius from the correct end: the 6 cm arc goes round A because  $AC = 6$  cm.

Do not forget the factor of 2 for the pairs of faces in a cuboid, and do not include the slanted 5 cm side in the triangle's area: the area of a right-angled triangle uses the two sides that meet at the right angle.

### 3 Scale drawings and bearings

#### Syllabus 4.3

A bearing is an angle measured *CLOCKWISE* from *NORTH*, written with three figures ( $065^\circ$ , not  $65^\circ$ ). The bearing back the other way differs by exactly  $180^\circ$ .

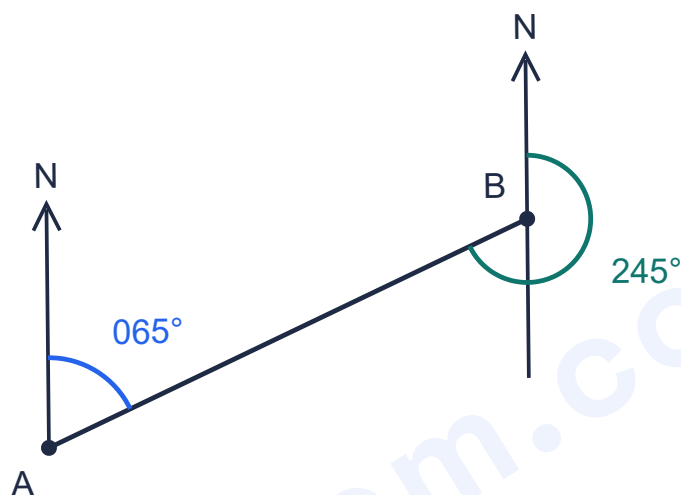


Fig. 1: the bearing of B from A is  $065^\circ$ ; the bearing of A from B is  $065^\circ + 180^\circ = 245^\circ$

#### Worked example: Map distance to real distance

A map has scale  $1 : 25\,000$ . Two churches are  $7.2\text{ cm}$  apart on the map. Find the real distance in kilometres.

1. Real distance  $= 7.2 \times 25\,000 = 180\,000\text{ cm}$ .
2.  $100\,000\text{ cm} = 1\text{ km}$ , so  $180\,000 \div 100\,000 = 1.8\text{ km}$ .

**Answer:**  $1.8\text{ km}$

## North line, clockwise, three figures.

- ✓ **Stand at the point after 'from':** that is where the north line goes.
- ✓ **Turn clockwise** from north to the line joining the points.
- ✓ **Three figures:**  $045^\circ$ , not  $45^\circ$ .
- ✓ **Back bearing:** add or subtract  $180^\circ$ .
- ✓ **Scales:** map to real, multiply; real to map, divide; convert units last.

## Watch out!

With a ratio scale both sides are in the SAME unit.  $1 : 25\ 000$  does not mean 1 cm to 25 000 m.

Multiply first to get centimetres, then convert to metres or kilometres.

Read the question's word order carefully. 'The bearing of A from B' means stand at B. Measuring at the wrong point gives the back bearing, which is wrong by exactly  $180^\circ$ .

Draw a NEW north line at every turning point. Measuring the second bearing from the first leg's line, instead of from north, is the classic error.

## 4 Similar shapes

### Syllabus 4.4

In similar shapes, corresponding angles are equal and corresponding lengths are in the same ratio. Scale factor  $k = \frac{\text{length on new shape}}{\text{matching length on old shape}}$ ; multiply any old length by  $k$  to get the new one.

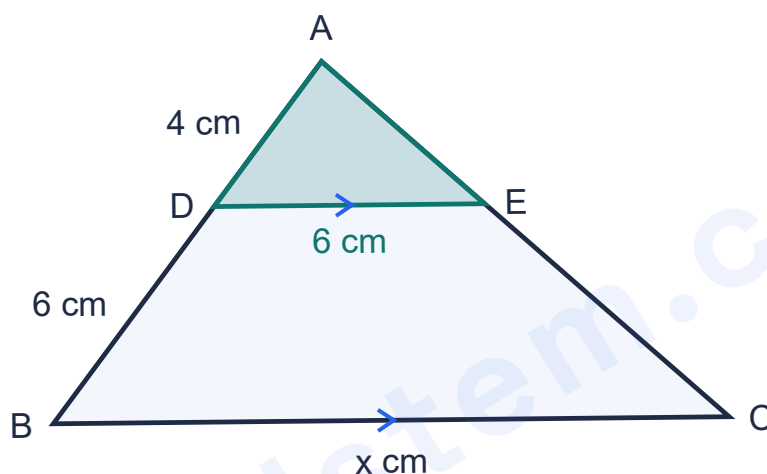


Fig. 1: DE is parallel to BC, so triangle ADE is similar to triangle ABC.  $AB = 4 + 6 = 10$  cm, so the scale factor is  $10 \div 4 = 2.5$  and  $x = 6 \times 2.5 = 15$  (drawn to scale)

### Worked example: Testing for similarity

Rectangle P is 4 cm by 6 cm. Rectangle Q is 10 cm by 15 cm. Rectangle R is 6 cm by 8 cm. Which of Q and R is similar to P?

1. P to Q:  $10 \div 4 = 2.5$  and  $15 \div 6 = 2.5$ . Same factor, so Q is similar to P.
2. P to R:  $6 \div 4 = 1.5$  but  $8 \div 6 = 1.33\dots$ . Different factors, so R is not similar to P.
3. All the angles are  $90^\circ$  in every rectangle, so for rectangles only the side ratios decide.

**Answer:** Q is similar to P (scale factor 2.5); R is not.

## Match, divide, multiply.

- ✓ **Match:** pair up corresponding sides (opposite equal angles). Redraw triangles separately if they overlap.
- ✓ **Divide:**  $k = \text{new} \div \text{old}$  from one known matching pair.
- ✓ **Multiply:**  $\text{new length} = \text{old length} \times k$ ;  
 $\text{old length} = \text{new length} \div k$ .
- ✓ **Nested triangles:** use whole sides from the shared vertex.
- ✓ **Reasons (Extended):** two pairs of equal angles, each with its geometric reason.

## Watch out!

Adding the same amount does not make shapes similar. A 4 by 6 rectangle with 3 cm added to each side becomes 7 by 9, and  $7 \div 4 \neq 9 \div 6$ . Similarity is about MULTIPLYING.

In nested triangles the scale factor uses the WHOLE side AB, not the extra piece DB. Using  $6 \div 4$  here gives 9 cm for BC, which is wrong.

Keep each ratio the same way round. Height over shadow for the girl equals height over shadow for the tree:  $\frac{h}{14} = \frac{1.8}{2.4}$ . Mixing a height with a shadow on one side gives nonsense.

## 5 Areas and volumes of similar shapes

### Syllabus 4.4

If lengths are multiplied by  $k$ , areas (including surface areas) are multiplied by  $k^2$  and volumes (and masses or capacities of the same material) by  $k^3$ . Always get back to the LENGTH factor  $k$  first.

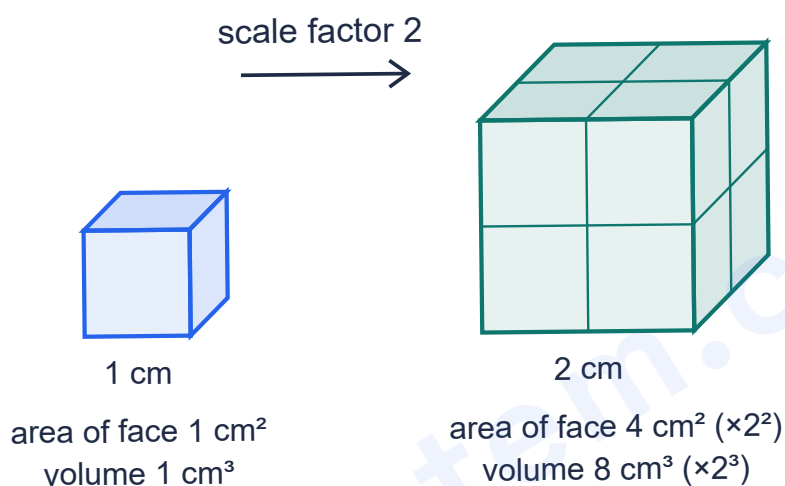


Fig. 1: enlarging a 1 cm cube by scale factor 2. Each face is made of  $2^2 = 4$  small squares and the cube of  $2^3 = 8$  small cubes

### Worked example: Area from a length ratio

Two similar triangles have corresponding sides in the ratio 2 : 5. The smaller triangle has area 12 cm<sup>2</sup>. Find the area of the larger.

1. Length factor  $k = \frac{5}{2} = 2.5$ .
2. Area factor  $k^2 = 2.5^2 = 6.25$ .
3. Larger area =  $12 \times 6.25 = 75$  cm<sup>2</sup>.

**Answer:** 75 cm<sup>2</sup>

### Get the length factor first.

- ✓ **Identify** what you are given: two lengths, two areas or two volumes.
- ✓ **Reduce to  $k$ :** divide, then square root (areas) or cube root (volumes).
- ✓ **Build up:**  $k$  for lengths,  $k^2$  for areas,  $k^3$  for volumes, capacity and mass.
- ✓ **Check direction:** bigger shape, bigger answer.
- ✓ **Units:**  $1 \text{ m}^2 = 10\,000 \text{ cm}^2$ ;  $1 \text{ km}^2 = 10^{10} \text{ cm}^2$ .

### Watch out!

Multiplying an area by the LENGTH factor is the most common error:  $12 \times 2.5 = 30 \text{ cm}^2$  is wrong. Areas need  $k^2$ .

Never go straight from an area ratio to a volume ratio by multiplying by the area factor. Find  $k$  first: from areas  $16 : 25$  the volume factor is  $(5/4)^3$ , not  $(25/16)$  and not  $(25/16)^2$ .

Mass scales with volume only for solids made of the same material. And a map scale of  $1 : 50\,000$  means AREAS shrink by  $50\,000^2$ , not by  $50\,000$ .

## 6 Line and rotational symmetry

### Syllabus 4.5

Count lines of symmetry by asking where a mirror could go; count the order of rotational symmetry by asking how many times the shape fits onto itself in one full turn. Every shape has order at least 1, and a regular  $n$ -sided polygon has  $n$  lines and order  $n$ .

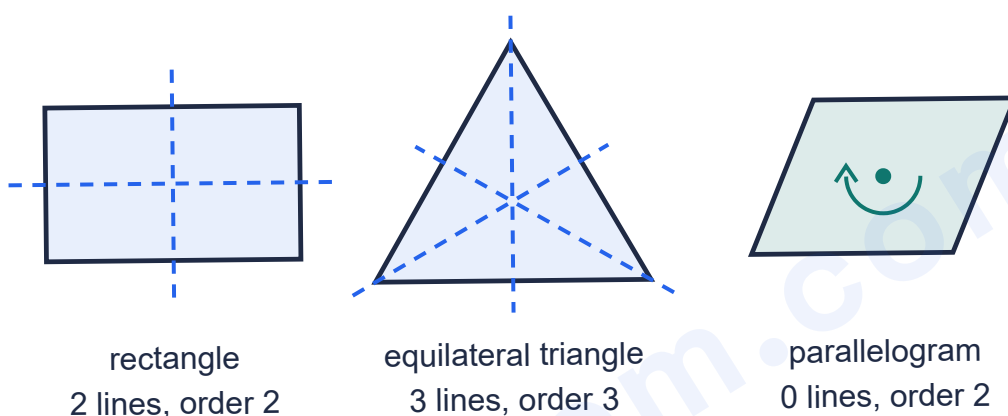


Fig. 1: lines of symmetry (dashed) and order of rotational symmetry. A parallelogram has no lines of symmetry but fits onto itself after a half turn about its centre

### Worked example: Symmetry tells you about the shape

Triangle ABC has exactly one line of symmetry, through A. Angle BAC =  $40^\circ$ . Find angle ABC.

1. One line of symmetry through A means the triangle is isosceles with  $AB = AC$ .
2. The mirror swaps B and C, so angle ABC = angle ACB.
3. Angles in a triangle add to  $180^\circ$ :  $180^\circ - 40^\circ = 140^\circ$  for the two base angles.
4. Angle ABC =  $140^\circ \div 2 = 70^\circ$ .

**Answer:**  $70^\circ$

### *Fold for lines, turn for order.*

- ✓ **Lines:** test every possible mirror: through vertices, through midpoints of sides.
- ✓ **Order:** count the fits in one full turn, including the start; minimum 1.
- ✓ **Angle of turn:**  $360^\circ \div \text{order}$ .
- ✓ **Regular polygons:**  $n$  sides,  $n$  lines, order  $n$ .
- ✓ **Solids (Extended):** planes along the length from the cross-section's lines, plus any plane across the middle.

### *Watch out!*

A rectangle's diagonals are NOT lines of symmetry: folding along a diagonal does not match the corners. A rhombus is the shape whose diagonals are lines of symmetry.

Order 0 does not exist. A shape with no rotational symmetry has order 1, because it always fits once, at the full turn.

For prisms, remember the extra plane that cuts across the middle of the length. For a cube, the six diagonal planes (through opposite edges) are easy to miss:  $3 + 6 = 9$ .

## 7 Angles: lines, parallel lines and triangles

### Syllabus 4.6

Angles at a point add to  $360^\circ$ ; on a straight line to  $180^\circ$ ; vertically opposite angles are equal; a triangle's angles add to  $180^\circ$  and a quadrilateral's to  $360^\circ$ . With parallel lines, corresponding and alternate angles are equal and co-interior angles add to  $180^\circ$ .

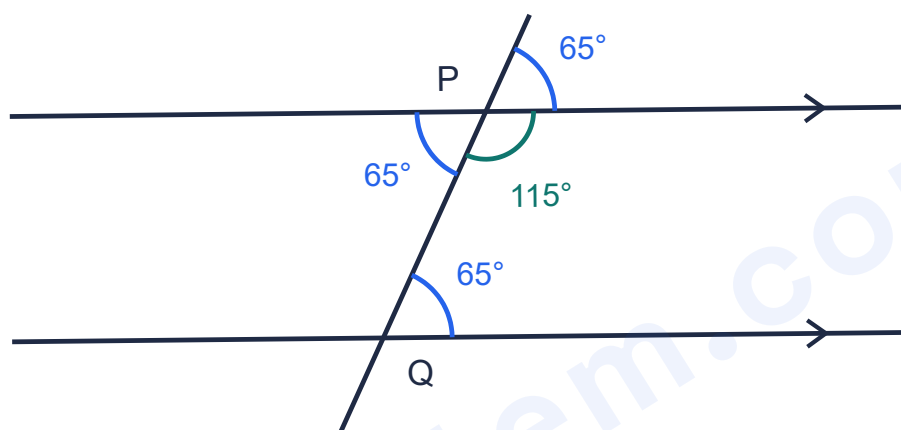


Fig. 1: a transversal crosses two parallel lines at  $65^\circ$  (drawn to scale). The  $65^\circ$  at Q is corresponding to the  $65^\circ$  above P, alternate to the  $65^\circ$  below-left of P, and co-interior with the  $115^\circ$  at P ( $65 + 115 = 180$ )

### Worked example: On a straight line

Three angles on a straight line are  $3x$ ,  $2x$  and  $40^\circ$ . Find  $x$ .

1. Angles on a straight line add to  $180^\circ$ :  $3x + 2x + 40 = 180$ .
2.  $5x = 140$ .
3.  $x = 28$ .

**Answer:**  $x = 28$  (the angles are  $84^\circ$ ,  $56^\circ$ ,  $40^\circ$ )

### Find an angle, name the fact.

- ✓ **Mark** every angle you find on the diagram as you go.
- ✓ **Look for** straight lines, points, crossings, triangles and parallel arrows.
- ✓ **Write equations** when angles are given in terms of  $x$ .
- ✓ **State the reason** in full words: 'alternate angles are equal'.
- ✓ **Check:** every triangle  $180^\circ$ , every straight line  $180^\circ$ .

### Watch out!

Write the reason in words, using the proper term: 'angles on a straight line add to  $180^\circ$ '. A correct number with no reason, or with 'because it is', often loses the reason mark.

Co-interior angles are NOT equal: they add to  $180^\circ$ . Setting  $2x + 15 = 3x + 15$  gives  $x = 0$ , a sure sign the wrong rule was used.

In an isosceles triangle, the equal angles are the ones *OPPOSITE* the equal sides, not the angle between them. Check which sides are marked equal before you halve anything.

## 8 Angles in polygons

### Syllabus 4.6

The exterior angles of any polygon add up to  $360^\circ$ , and at each vertex interior + exterior =  $180^\circ$ . The interior angles of an  $n$ -sided polygon add up to  $(n - 2) \times 180^\circ$ . For a regular polygon, each exterior angle is  $\frac{360^\circ}{n}$ .

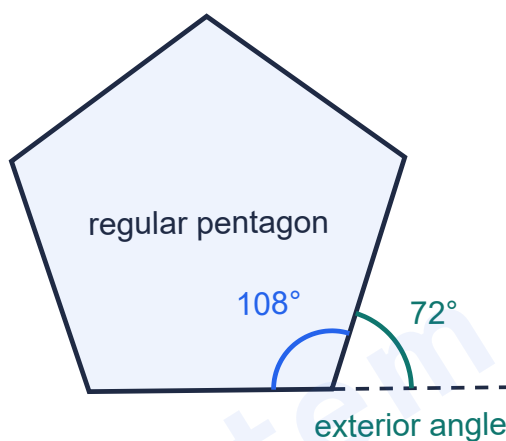


Fig. 1: a regular pentagon drawn to scale. Exterior angle =  $360^\circ \div 5 = 72^\circ$ , so interior angle =  $180^\circ - 72^\circ = 108^\circ$

### Worked example: A missing angle (Extended: irregular polygon)

Four of the interior angles of a pentagon are  $100^\circ$ ,  $120^\circ$ ,  $95^\circ$  and  $110^\circ$ . Find the fifth angle.

1. Angle sum of a pentagon =  $(5 - 2) \times 180^\circ = 540^\circ$ .
2. The four known angles total  $100 + 120 + 95 + 110 = 425^\circ$ .
3. Fifth angle =  $540^\circ - 425^\circ = 115^\circ$ .

**Answer:**  $115^\circ$

### *Exterior first, then interior.*

- ✓ **Angle sum:**  $(n - 2) \times 180^\circ$ .
- ✓ **Exterior sum:** always  $360^\circ$ .
- ✓ **Regular:** exterior =  $360^\circ \div n$ , interior =  $180^\circ - \text{exterior}$ .
- ✓ **Number of sides:**  $n = 360^\circ \div \text{exterior}$ ; it must be a whole number.
- ✓ **Shapes at a point:** interior angles add to  $360^\circ$ .

### *Watch out!*

The formula is  $(n - 2) \times 180^\circ$ , not  $n \times 180^\circ$ . Splitting from one vertex gives  $n - 2$  triangles; a pentagon makes 3 triangles, not 5.

Dividing 360 by the INTERIOR angle is a common slip:  $360 \div 156$  is not a number of sides. Convert to the exterior angle first.

Do not use the exterior angle when shapes meet at a point. The corners that fill the space round a point are the INTERIOR angles.

## 9 Circle theorems I: semicircles and tangents

### Syllabus 4.7

The angle in a semicircle is  $90^\circ$ : a triangle drawn on a diameter with its third corner on the circle has a right angle at that corner. A tangent meets the radius at its point of contact at  $90^\circ$ .

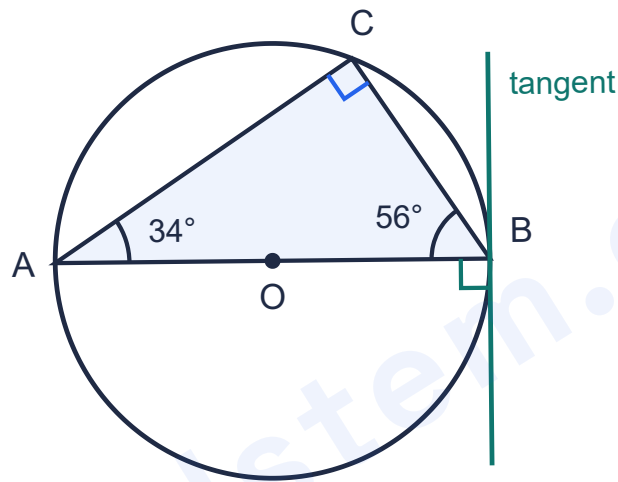


Fig. 1:  $AB$  is a diameter, so angle  $ACB = 90^\circ$  (angle in a semicircle). The tangent at  $B$  is perpendicular to the radius  $OB$ . Drawn to scale

### Worked example: Using the right angle

In Fig. 1,  $AB$  is a diameter and angle  $CAB = 34^\circ$ . Find angle  $ABC$ .

1. Angle  $ACB = 90^\circ$  (angle in a semicircle).
2. Angles in triangle  $ABC$  add to  $180^\circ$ : angle  $ABC = 180^\circ - 90^\circ - 34^\circ$ .
3. Angle  $ABC = 56^\circ$ .

**Answer:**  $56^\circ$

### Spot the right angle, then use the triangle.

- ✓ **Diameter?** Right angle at the circumference (angle in a semicircle).
- ✓ **Tangent?** Right angle where it meets the radius.
- ✓ **Radii?** Isosceles triangle, equal base angles.
- ✓ **Finish** with the angle sum, Pythagoras or trigonometry.
- ✓ **Reasons** in syllabus words for every angle.

### Watch out!

The right angle is at  $C$ , the point on the circumference, NOT at the centre and not at  $A$  or  $B$ . It only holds when  $AB$  really is a diameter, so check the chord passes through  $O$ .

$OP$ , the line from the centre to the outside point, is the hypotenuse, not the tangent. Subtract squares to find the tangent:  $PT^2 = OP^2 - OT^2$ .

Write the circle reason in the syllabus words. 'Because it touches the circle' does not earn the mark; 'angle between tangent and radius =  $90^\circ$ ' does.

## 10 Circle theorems II: centre, segments, cyclic quadrilaterals

### Syllabus 4.8

Angle at the centre =  $2 \times$  angle at the circumference (same arc). Angles in the same segment are equal. Opposite angles of a cyclic quadrilateral add to  $180^\circ$ . The angle between a tangent and a chord equals the angle in the alternate segment. Tangents from one point are equal, and the perpendicular from the centre bisects a chord.

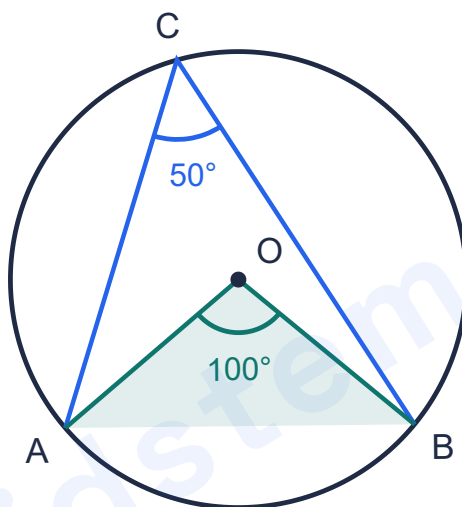


Fig. 1: arc  $AB$  subtends  $100^\circ$  at the centre  $O$  and  $50^\circ$  at  $C$  on the circumference: the angle at the centre is twice the angle at the circumference. Drawn to scale

### Worked example: Centre and circumference

$A$ ,  $B$  and  $C$  lie on a circle with centre  $O$ . Angle  $AOB = 124^\circ$  and  $C$  is on the major arc. Find angle  $ACB$ . Another point  $D$  lies on the major arc. Find angle  $ADB$ .

1. Angle  $ACB = 124^\circ \div 2 = 62^\circ$  (angle at the centre is twice the angle at the circumference).
2.  $D$  and  $C$  are in the same segment, standing on arc  $AB$ .
3. Angle  $ADB = \text{angle } ACB = 62^\circ$  (angles in the same segment are equal).

**Answer:** Angle  $ACB = 62^\circ$ , angle  $ADB = 62^\circ$

## Which arc? Which theorem?

- ✓ **Centre involved?** Angle at centre =  $2 \times$  angle at circumference on the same arc.
- ✓ **Two angles on the same arc?** Same segment: equal.
- ✓ **Four points on the circle?** Opposite angles add to  $180^\circ$ .
- ✓ **Tangent and chord?** Alternate segment theorem; tangent and radius  $90^\circ$ .
- ✓ **Chord or two tangents?** Perpendicular from centre bisects; tangents from a point are equal.
- ✓ **Reasons** in syllabus words for every step.

## Watch out!

Halve the angle at the centre to get the circumference angle, do not double it. And check which arc each angle stands on: if the angle at the circumference is obtuse, the matching centre angle is reflex.

Only **OPPOSITE** angles add to  $180^\circ$ . Adjacent angles of a cyclic quadrilateral have no special rule, and a quadrilateral whose vertices are not all on the circle has no such rule at all.

The equal angle is in the **ALTERNATE** segment, across the chord from the tangent angle. Matching it with an angle in the same segment as the tangent angle is the usual error.

The right angles in the tangent kite are at A and B (where the tangents touch), not at P or O. Angle AOB and angle APB are supplementary, not equal.