

Mensuration

IGCSE Maths 0580 · Topic 5 revision notes

Everything from the 6 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Units of measure and conversions

2. Area and perimeter of 2D shapes

3. Circles, arcs and sectors

4. Surface area and volume: prisms and cylinders

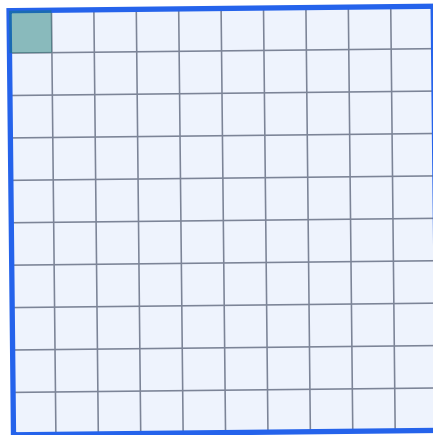
5. Surface area and volume: cones, pyramids and spheres

6. Compound shapes and parts of shapes

1 Units of measure and conversions

Syllabus 5.1

Convert a length first, then square it for area or cube it for volume. Since $1 \text{ m} = 100 \text{ cm}$, $1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$ and $1 \text{ m}^3 = 100^3 = 1\,000\,000 \text{ cm}^3$. For capacity, $1 \text{ cm}^3 = 1 \text{ ml}$ and $1000 \text{ cm}^3 = 1 \text{ litre}$.



$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ cm}^2 = 10 \times 10 \\ = 100 \text{ mm}^2$$

each small square
is 1 mm^2

Fig. 1: a 1 cm square is 10 mm by 10 mm , so it holds 100 squares of 1 mm^2 , not 10

Worked example: Big unit to small unit

Convert (a) 3.75 km to metres (b) 0.6 litres to millilitres.

1. (a) Metres are smaller than kilometres, so there are more of them: multiply by 1000.
2. $3.75 \times 1000 = 3750$.
3. (b) Millilitres are smaller, so multiply by 1000: $0.6 \times 1000 = 600$.

Answer: (a) 3750 m (b) 600 ml

Length factor first, then square or cube it.

- ✓ **Direction:** to a smaller unit, multiply; to a bigger unit, divide.
- ✓ **Length:** $10 \text{ mm} = 1 \text{ cm}$, $100 \text{ cm} = 1 \text{ m}$, $1000 \text{ m} = 1 \text{ km}$.
- ✓ **Area:** square the length factor, e.g. $1 \text{ m}^2 = 10\,000 \text{ cm}^2$.
- ✓ **Volume:** cube it, e.g. $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$.
- ✓ **Capacity:** $1 \text{ cm}^3 = 1 \text{ ml}$, $1000 \text{ cm}^3 = 1 \text{ l}$, $1 \text{ m}^3 = 1000 \text{ l}$.
- ✓ **Mixed units:** change everything to one unit before calculating.

Watch out!

Check the answer makes sense: 3.75 km is a long walk, so the number of metres must be LARGE (3750), not 0.003 75. If the number got smaller when you moved to a smaller unit, you divided when you should have multiplied.

1 m^2 is NOT 100 cm^2 . A square metre is 100 cm by 100 cm, which is $10\,000 \text{ cm}^2$. Using 100 is the most common lost mark in this topic.

1 m^3 is 1000 litres, not 100 and not 1 million. Go through cm^3 if unsure: $1\,000\,000 \text{ cm}^3 \div 1000 = 1000 \text{ litres}$.

When the answer must be a whole number of objects (glasses, bottles, tiles), decide whether to round down (how many can be FILLED) or up (how many are NEEDED). Do not just round to the nearest.

2 Area and perimeter of 2D shapes

Syllabus 5.2

Area always uses a length and the PERPENDICULAR height: rectangle lw , triangle $\frac{1}{2}bh$, parallelogram bh , trapezium $\frac{1}{2}(a+b)h$. Only the triangle formula is in the List of formulas, so learn the other three. Perimeter is just the sum of the sides.

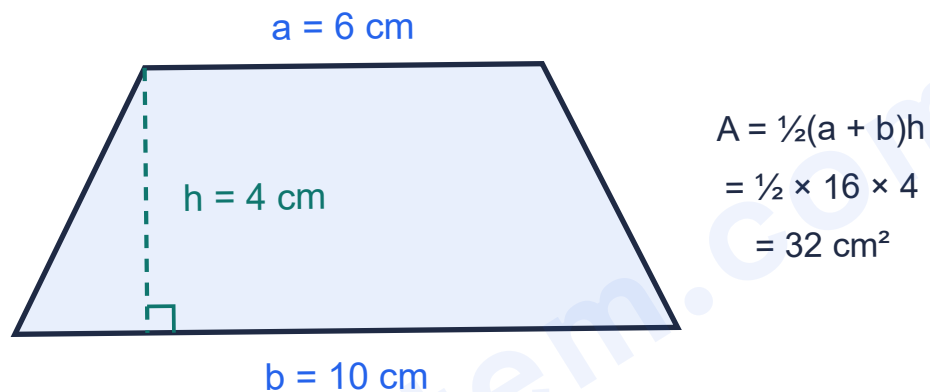


Fig. 1: a trapezium with parallel sides 6 cm and 10 cm, 4 cm apart (drawn to scale); the height is measured at right angles to the parallel sides

Worked example: Rectangle

A rectangle is 12 cm long and 7.5 cm wide. Find its area and its perimeter.

1. Area $= l \times w = 12 \times 7.5 = 90$.
2. Perimeter = sum of all four sides $= 12 + 7.5 + 12 + 7.5 = 39$.

Answer: Area 90 cm^2 , perimeter 39 cm

Perpendicular height, every time.

- ✓ **Rectangle:** $A = lw$.
- ✓ **Triangle:** $A = \frac{1}{2}bh$ (the only one given in the exam).
- ✓ **Parallelogram:** $A = bh$.
- ✓ **Trapezium:** $A = \frac{1}{2}(a + b)h$, a and b parallel.
- ✓ **Perimeter:** add every side, including the sloping ones.
- ✓ **Backwards:** substitute what you know and solve for the missing length.

Watch out!

Using the sloping side instead of the perpendicular height: $\frac{1}{2} \times 10 \times 7 = 35$ is wrong. The height is always the one that meets the base at a right angle.

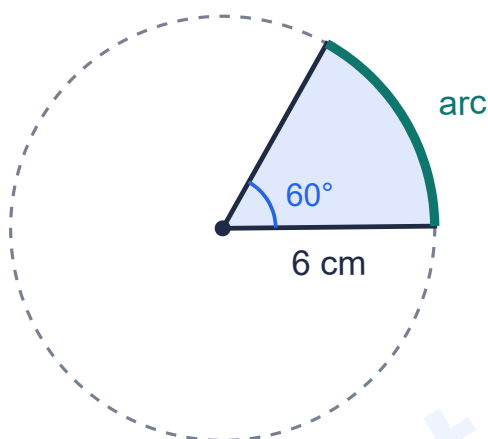
In a trapezium the two sides you add must be the PARALLEL ones, and h is the distance between them. Multiplying all three numbers together, or forgetting the half, are the usual slips.

Keep units straight: perimeter is a length (cm), area is in square units (cm^2). Writing 40 cm^2 for a perimeter loses the unit mark.

3 Circles, arcs and sectors

Syllabus 5.3

Circumference $C = 2\pi r$ and area $A = \pi r^2$ are given in the List of formulas. For a sector with angle θ : arc length $= \frac{\theta}{360} \times 2\pi r$ and sector area $= \frac{\theta}{360} \times \pi r^2$. These two are NOT printed: build them from the fraction of the circle.



$$\begin{aligned}\text{arc} &= 60/360 \times 2\pi \times 6 \\ &= 2\pi \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{sector} &= 60/360 \times \pi \times 6^2 \\ &= 6\pi \text{ cm}^2\end{aligned}$$

Fig. 1: a 60 degree sector of a circle of radius 6 cm is one sixth of the circle, so its arc is one sixth of the circumference and its area one sixth of the circle's area

Worked example: From the radius

A circle has radius 4.5 cm. Find (a) its circumference (b) its area.

1. (a) $C = 2\pi r = 2 \times \pi \times 4.5 = 9\pi = 28.27\dots$

2. (b) $A = \pi r^2 = \pi \times 4.5^2 = 20.25\pi = 63.61\dots$

Answer: (a) 28.3 cm (b) 63.6 cm²

Find the fraction, then take that much of the circle.

- ✓ **Given:** $C = 2\pi r$ and $A = \pi r^2$ (List of formulas).
- ✓ **Radius:** halve a diameter before using the formulas.
- ✓ **Fraction:** sector angle $\div 360$.
- ✓ **Arc and sector:** fraction $\times 2\pi r$ and fraction $\times \pi r^2$.
- ✓ **Perimeter of a sector:** arc + 2 radii.
- ✓ **Exact:** "in terms of π " means keep π as a symbol.

Watch out!

Putting the diameter into πr^2 : a circle of diameter 10 cm has area 25π , not 100π . Halve the diameter before you square.

The perimeter of a sector is NOT just the arc. Add the two radii (for a semicircle, the diameter). Leaving them out is the examiner's favourite lost mark.

When asked for a major sector, use $360 - \theta$, not θ . Read which region is shaded before you start.

4 Surface area and volume: prisms and cylinders

Syllabus 5.4

Volume of a prism = cross-section area $\times$ length ($V = Al$), so a cylinder has $V = \pi r^2 h$. Its curved surface area is $2\pi r h$. All three are in the List of formulas; the TOTAL surface area of a cylinder, $2\pi r h + 2\pi r^2$, is not, so add the two circular ends yourself.

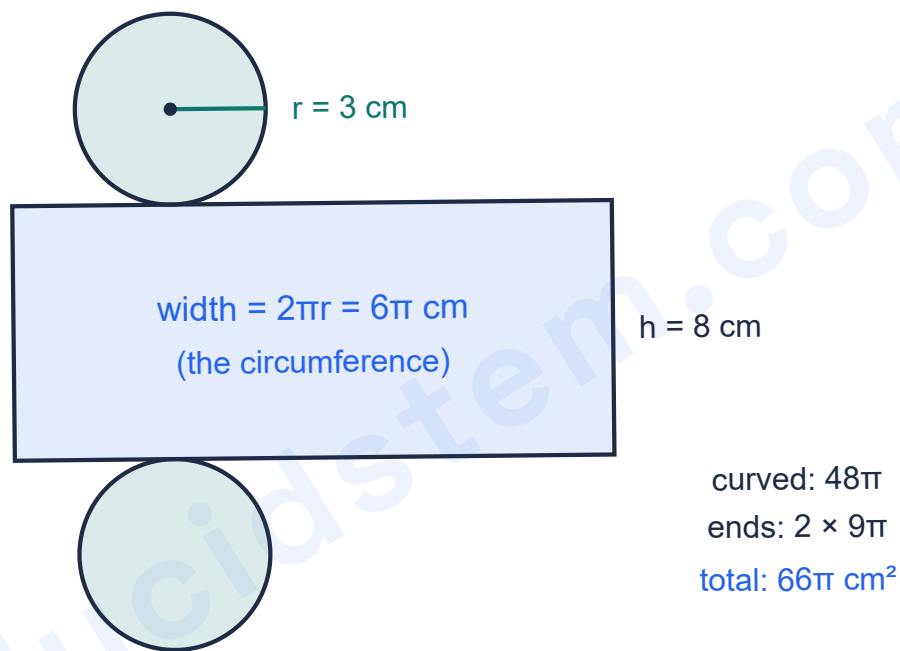


Fig. 1: the net of a cylinder of radius 3 cm and height 8 cm, drawn to scale; the curved surface unrolls to a $6\pi\text{ cm}$ by 8 cm rectangle

Worked example: Cuboid

A cuboid measures 8 cm by 5 cm by 3 cm. Find its volume and its surface area.

1. Volume = $8 \times 5 \times 3 = 120$.
2. Faces come in three pairs: $8 \times 5 = 40$, $8 \times 3 = 24$, $5 \times 3 = 15$.
3. Surface area = $2(40 + 24 + 15) = 158$.

Answer: Volume 120 cm^3 , surface area 158 cm^2

Cross-section first, then the length.

- ✓ **Prism:** $V = Al$, A = cross-section area (given).
- ✓ **Cylinder:** $V = \pi r^2 h$ and curved surface $2\pi r h$ (both given).
- ✓ **Total surface area:** add every face; for a cylinder add $2\pi r^2$ for the ends.
- ✓ **Open containers:** leave out the missing face.
- ✓ **Backwards:** substitute the volume and solve; round only at the end.
- ✓ **Capacity:** $1000 \text{ cm}^3 = 1 \text{ litre}$.

Watch out!

Surface area means EVERY face. In a triangular prism it is easy to count the two triangles and forget one of the three rectangles, especially the sloping one.

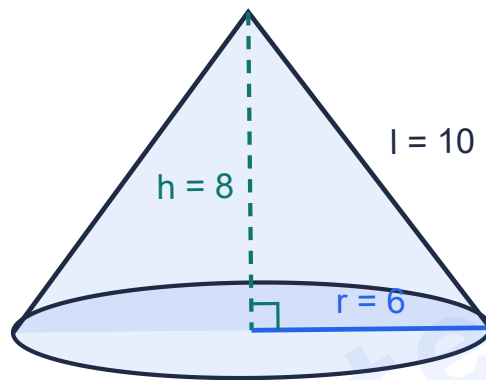
The List of formulas gives the CURVED surface area $2\pi r h$ only. For a closed cylinder add $2\pi r^2$ for the ends; for an open tin (no lid) add just one πr^2 .

Rounding the radius or an area too early changes the third significant figure. Store values in the calculator and round once at the end.

5 Surface area and volume: cones, pyramids and spheres

Syllabus 5.4

Pyramid $V = \frac{1}{3}Ah$; cone $V = \frac{1}{3}\pi r^2 h$ and curved surface $\pi r l$; sphere $V = \frac{4}{3}\pi r^3$ and surface $4\pi r^2$. All are in the List of formulas. The cone's curved surface uses the SLANT height l , where $l^2 = r^2 + h^2$; that relationship is not given.



$$\begin{aligned} l^2 &= r^2 + h^2 \\ &= 36 + 64 = 100 \\ l &= 10 \text{ cm} \end{aligned}$$

Fig. 1: a cone of radius 6 cm and vertical height 8 cm (drawn to scale); the slant height is 10 cm by Pythagoras

Worked example: Square-based pyramid

A pyramid has a square base of side 9 cm and a vertical height of 10 cm. Find its volume.

1. Base area $A = 9 \times 9 = 81 \text{ cm}^2$.

2. $V = \frac{1}{3} \times 81 \times 10 = 270$.

Answer: 270 cm^3

Pick the formula, then the length it needs.

- ✓ **Pyramid and cone volume:** $\frac{1}{3}$ of base area $\times$ vertical height.
- ✓ **Cone curved surface:** $\pi r l$ with slant height $l = \sqrt{r^2 + h^2}$.
- ✓ **Sphere:** $V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$.
- ✓ **Solid shapes:** add flat faces (cone base πr^2 , hemisphere face πr^2).
- ✓ **Melting and recasting:** the volume stays the same.

Watch out!

The h in $\frac{1}{3}Ah$ is the VERTICAL height from the base to the apex, not the sloping height of a face. Forgetting the $\frac{1}{3}$ gives the volume of the prism instead.

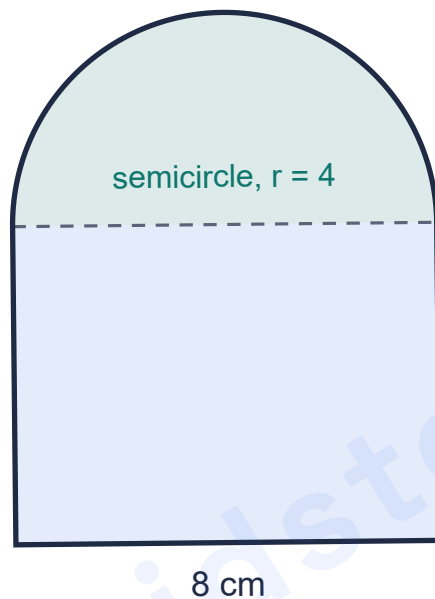
Putting the vertical height into $\pi r l$: for the cone above, $\pi \times 5 \times 12$ is wrong. The curved surface needs the slant height 13.

The total surface area of a solid hemisphere is $2\pi r^2 + \pi r^2 = 3\pi r^2$. Halving $4\pi r^2$ and stopping forgets the flat face.

6 Compound shapes and parts of shapes

Syllabus 5.5

Area and volume: add the pieces, subtract the holes. Perimeter and surface area: include only the OUTSIDE edges and faces. A line or face where two pieces join is inside the shape, so it does not count.



Area
 $48 + 8\pi = 73.1 \text{ cm}^2$

Perimeter
 $6 + 8 + 6 + 4\pi$
 $= 32.6 \text{ cm}$

Fig. 1: a window made of an 8 cm by 6 cm rectangle and a semicircle of diameter 8 cm (drawn to scale); the dashed join is not part of the perimeter

Worked example: An L-shape

A 10 cm by 8 cm rectangle has a 4 cm by 5 cm rectangle cut from one corner, leaving an L-shape. Find its area and perimeter.

1. Area: whole rectangle minus the cut-out $= 10 \times 8 - 4 \times 5 = 80 - 20 = 60$.
2. Perimeter: the two inner edges (4 cm and 5 cm) replace the two missing pieces of the outer edges exactly.
3. So the perimeter is the same as the rectangle's: $2(10 + 8) = 36$.

Answer: Area 60 cm^2 , perimeter 36 cm

Split into known shapes, then add or subtract.

- ✓ **Sketch** the pieces and label every length you know.
- ✓ **Area and volume:** add the pieces, subtract the holes.
- ✓ **Perimeter:** outside edges only, never the joins.
- ✓ **Surface area:** outside faces only; joined faces disappear.
- ✓ **Frustum (Extended):** big cone minus small cone.
- ✓ **Accuracy:** keep π and unrounded values until the end.

Watch out!

Do not include the join. In Fig. 1 the 8 cm line under the semicircle is inside the window, so adding it (40.6 cm) is wrong.

For a ring, subtract the AREAS, not the radii: $\pi(7 - 4)^2 = 9\pi$ is wrong. Square each radius first: $\pi(7^2 - 4^2) = 33\pi$.

For surface area, leave out every face where two parts are joined. Counting the circle between the cylinder and the hemisphere adds an extra 16π .

You cannot use the frustum's own height in a single cone formula. Always work with the two complete cones and subtract.