Number

IGCSE Maths 0580 · Topic 1 revision notes

Everything from the 19 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Types of number: primes, factors, multiples, rational and irrational
2. Sets, set notation and Venn diagrams
3. Squares, cubes and roots
4. Fractions, decimals and percentages
5. Ordering quantities and inequality symbols
6. The four operations: fractions, negatives and order of operations
7. Indices: the laws of indices
8. Standard form
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13. Percentages: change, reverse percentages
14. Simple and compound interest
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1 Types of number: primes, factors, multiples, rational and irrational

Syllabus 1.1

Every whole number greater than 1 can be written as a product of primes in exactly one way. Write both numbers as prime products: the HCF takes the primes they share (lowest powers), the LCM takes every prime that appears (highest powers).

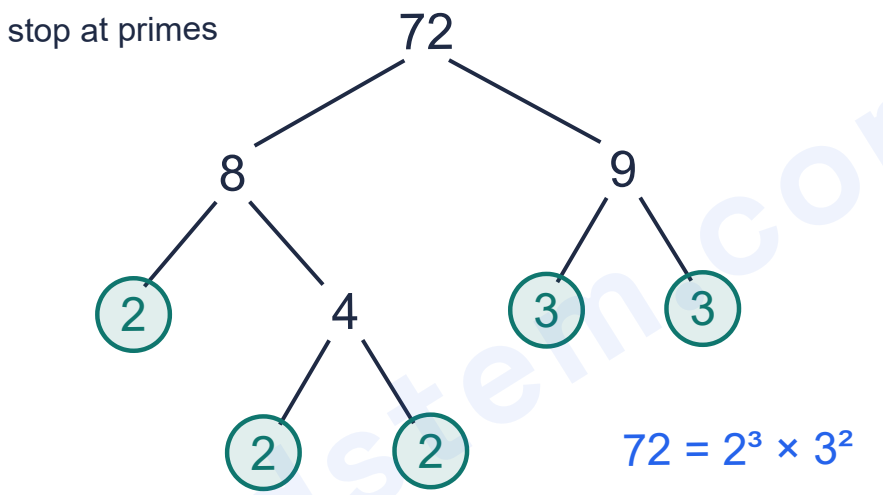

Fig. 1: a factor tree for 72. Split until every branch ends in a prime (circled), then collect them: 72 = 2 × 2 × 2 × 3 × 3

Worked example: Sorting a list

From the list $1,\ 2,\ 9,\ 15,\ 27,\ 31,\ 64$ write down (a) the prime numbers (b) the square numbers (c) the cube numbers.

1. (a) Test each number for factors other than 1 and itself: 9, 15, 27 and 64 all have extra factors; 1 has only one factor so it is not prime.
2. So the primes are 2 and 31.
3. (b) $1 = 1^2$, $9 = 3^2$, $64 = 8^2$.
4. (c) $1 = 1^3$, $27 = 3^3$, $64 = 4^3$.

Answer: (a) 2, 31 (b) 1, 9, 64 (c) 1, 27, 64

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Break it into primes, then build it back up.

- ✓ **Prime:** exactly two factors. 1 is not prime; 2 is the only even prime.
- ✓ **Prime factors:** divide by the smallest prime again and again; write the answer with indices.
- ✓ **HCF:** primes in both numbers, at the lowest power.
- ✓ **LCM:** every prime that appears, at the highest power.
- ✓ **Rational:** can be written as $\frac{a}{b}$; simplify roots before deciding.
- ✓ **Reciprocal:** $1 \div$ the number; make mixed numbers improper first.

Watch out!

1 is not a prime number, and 2 is the only even prime. A list of primes starting $1, 3, 5, 7, 9$ loses marks twice: 1 and 9 are not prime, and 2 is missing.

$72 = 8 \times 9$ is true, but it is not a product of PRIMES: 8 and 9 both split further. Keep going until every factor is prime, then use index notation: $2^3 \times 3^2$.

Do not mix the rules. The HCF can never be bigger than the smaller number, and the LCM can never be smaller than the bigger one. If your HCF of 84 and 90 is 1260, you have swapped them.

$\sqrt{9}$ looks irrational because it has a root sign, but $\sqrt{9} = 3$, which is rational. Always simplify first. And the reciprocal of $1\frac{2}{3}$ is not $1\frac{3}{2}$: convert to an improper fraction before turning it over.

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2 Sets, set notation and Venn diagrams

Syllabus 1.2

Fill a Venn diagram from the middle outwards: the intersection first, then the 'only' regions, then the outside. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, because the overlap is counted twice otherwise.

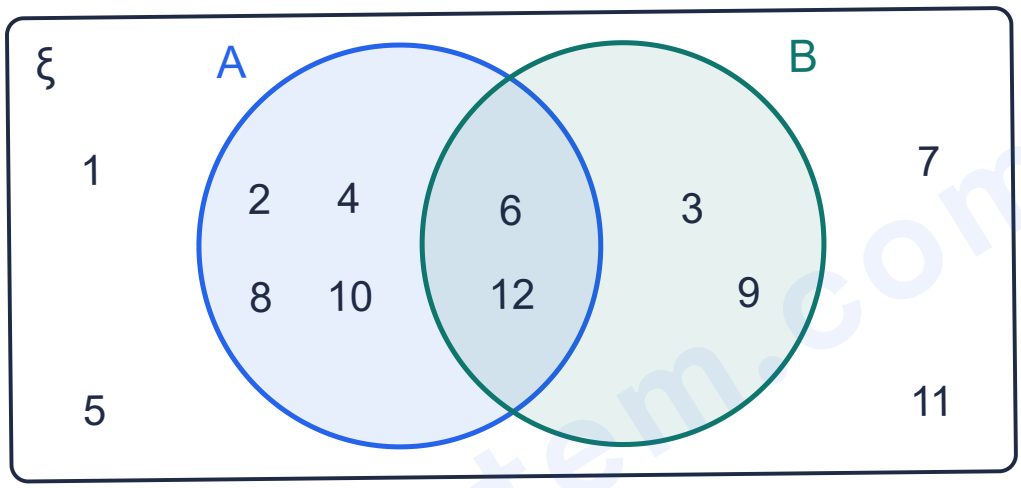

Fig. 1: ξ = {1, 2, …, 12}, A = {even numbers}, B = {multiples of 3}. The overlap $A \cap B$ = {6, 12}; the numbers outside both circles form $(A \cup B)'$ = {1, 5, 7, 11}

Worked example: Listing sets

$\xi = \{1, 2, 3, \ldots, 12\}$, $A = \{\text{even numbers}\}$, $B = \{\text{multiples of } 3\}$. List $A \cap B$, $A \cup B$ and A', and find $n(A \cup B)$.

1. $A = \{2, 4, 6, 8, 10, 12\}$ and $B = \{3, 6, 9, 12\}$.
2. Intersection: the numbers in both lists, $A \cap B = \{6, 12\}$.
3. Union: every number in either list, written once: $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$, so $n(A \cup B) = 8$.
4. Complement: the numbers in ξ not in A, $A' = \{1, 3, 5, 7, 9, 11\}$.

Answer: $A \cap B = \{6, 12\}$, $A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12\}$, $A' = \{1, 3, 5, 7, 9, 11\}$, $n(A \cup B) = 8$ (see Fig. 1)

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Fill the overlap first, then subtract.

- ✓ **List** each set in full before combining them.
- ✓ **Union** $\cup$: either or both, each element once. **Intersection** $\cap$: both.
- ✓ **Complement** A': everything in ξ not in A.
- ✓ **From counts:** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- ✓ **Three sets:** centre first, then the pairs, then the 'only' regions, then outside. Check the total.

Watch out!

In a union, list a shared element only ONCE. Writing $A \cup B = \{2, 4, 6, 8, 10, 12, 3, 6, 9, 12\}$ double-counts 6 and 12 and gives the wrong $n(A \cup B)$.

Do not write $n(A) = 18$ in the 'A only' region. 18 is the whole circle; the 'A only' region is $18 - 7 = 11$. Writing 18 there makes the diagram add up to more than 30.

$\varnothing$ means the empty set; $\{0\}$ is NOT empty, it has one element (the number 0). And $n(\varnothing) = 0$.

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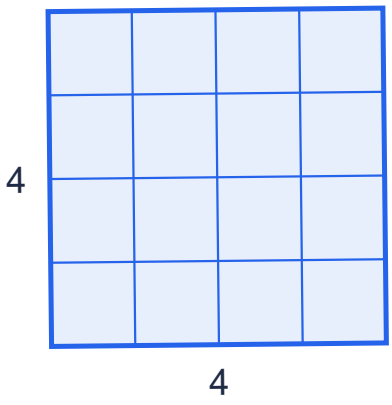
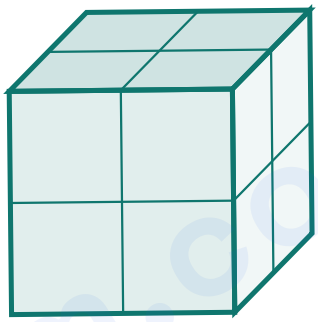
3 Squares, cubes and roots

Syllabus 1.3

A root undoes a power: $\sqrt{169} = 13$ because $13^2 = 169$, and $\sqrt[3]{64} = 4$ because $4^3 = 64$. A square root of a positive number is taken as positive; a cube root keeps the sign, so $\sqrt[3]{-125} = -5$.





$4^2 = 16$, $\sqrt{16} = 4$ $\quad$ $2^3 = 8$, $\sqrt[3]{8} = 2$

Fig. 1: a square of side 4 is made of $4^2 = 16$ unit squares; a cube of edge 2 is made of $2^3 = 8$ unit cubes. The root recovers the side from the total

Worked example: Recall

Write down the value of (a) $\sqrt{169}$ (b) $\sqrt{0.49}$.

1. (a) $13^2 = 169$, so $\sqrt{169} = 13$.
2. (b) $0.7^2 = 0.49$ (since $7^2 = 49$ and 0.7×0.7 has two decimal places).

Answer: (a) 13 (b) 0.7

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Know the facts, then let the root undo the power.

- ✓ **Recall:** squares 1 to 15 (up to 225); cubes of 1, 2, 3, 4, 5 and 10.
- ✓ **Decimals:** $0.3^2 = 0.09$ (2 d.p.), $0.2^3 = 0.008$ (3 d.p.).
- ✓ **Signs:** $\sqrt[3]{-64} = -4$; $(-3)^2 = 9$ but $-3^2 = -9$.
- ✓ **Under a root:** work out everything underneath first; the root does not split over $+$ or $-$.
- ✓ **Trapping:** find the squares (or cubes) either side to locate a root.

Watch out!

-3^2 means $-(3^2) = -9$, but $(-3)^2 = 9$. On a calculator, put brackets round a negative number before squaring it.

Cubing is not multiplying by 3: $4^3 = 4 \times 4 \times 4 = 64$, not 12. In the same way $\sqrt[3]{27} = 3$, not 9.

$\sqrt{12.5^2 - 3.5^2}$ is NOT $12.5 - 3.5 = 9$. A root does not split over a plus or minus sign; work out the whole expression underneath first.

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4 Fractions, decimals and percentages

Syllabus 1.4

Percent means 'out of 100'. To convert, go through the decimal: fraction to decimal by dividing top by bottom, decimal to percentage by multiplying by 100. Always cancel a fraction to its simplest form by dividing top and bottom by their HCF.

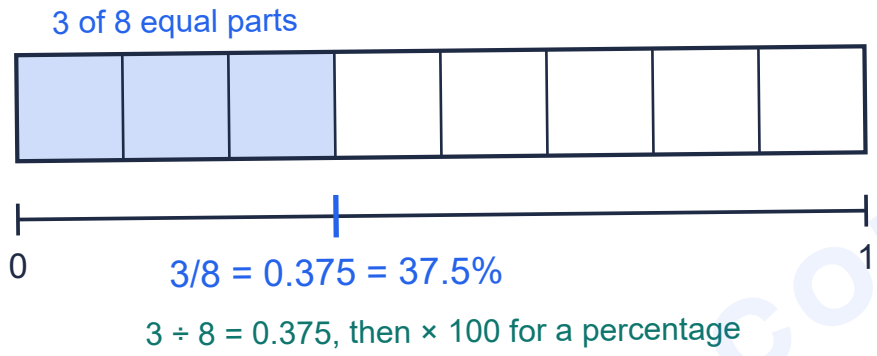

Fig. 1: three of eight equal parts is the fraction 3/8; on the number line it sits at 0.375, which is 37.5 per cent of the whole

Worked example: Fraction to decimal and percentage

Write $\frac{3}{8}$ as (a) a decimal (b) a percentage.

1. (a) $3 \div 8$: 8 into 3.000 gives 0.375.
2. (b) $0.375 \times 100 = 37.5$.

Answer: (a) 0.375 (b) 37.5%

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Go through the decimal, then cancel.

- ✓ **Fraction to decimal:** top ÷ bottom.
- ✓ **Decimal to percentage:** $\times 100$; percentage to decimal: $\div 100$.
- ✓ **Decimal to fraction:** place value (tenths, hundredths, ...), then cancel by the HCF.
- ✓ **Mixed to improper:** whole × bottom + top, over the same bottom.
- ✓ **Recurring (Extended):** let x = the decimal, multiply by a power of 10 to line up the repeats, subtract.

Watch out!

0.4 is 40%, not 4%. Multiplying by 100 moves the digits TWO places. $4\% = 0.04$.

A percentage over 100 is fine: 165% is more than one whole. Do not 'correct' it to 16.5% or 1.65%; as a decimal it is 1.65.

$0.1\dot{7}$ (0.1777...) and $0.\dot{1}\dot{7}$ (0.171 717...) are different numbers. Read the dots carefully before choosing what to multiply by, and show the subtraction: 'show that' questions give no marks for a calculator answer.

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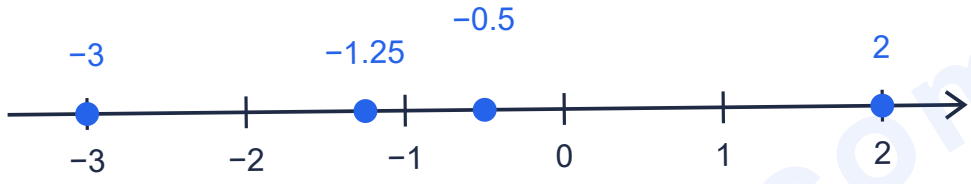
5 Ordering quantities and inequality symbols

Syllabus 1.5

To order mixed quantities, convert every one into the same form (usually a decimal to at least 3 or 4 places, or the same unit), compare, then write the answer using the ORIGINAL forms in the order asked for.



← smaller (further left) larger (further right) →

Fig. 1: on a number line, further right means larger, so $-3 < -1.25 < -0.5 < 2$ even though 3 looks bigger than 1.25

Worked example: Fractions, decimals and percentages

Write in order of size, smallest first: $\frac{5}{8},\ 0.62,\ 61\%,\ 0.6$.

1. Convert each to a decimal: $\frac{5}{8} = 0.625,\ 61\% = 0.61$.
2. Line up to 3 places: $0.625,\ 0.620,\ 0.610,\ 0.600$.
3. Smallest first: $0.600,\ 0.610,\ 0.620,\ 0.625$.
4. Answer in the original forms.

Answer: $0.6,\ 61\%,\ 0.62,\ \frac{5}{8}$

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One form, enough decimal places, original answers.

- ✓ **Convert** everything to decimals (or to one unit).
- ✓ **Pad** to the same number of decimal places, 4 if values are close.
- ✓ **Negatives:** further from zero means smaller.
- ✓ **Symbols:** the wide end of $<$ or $>$ faces the larger number; $\leq$ and $\geq$ include the end value.
- ✓ **Answer** in the original forms, in the order the question asks.

Watch out!

More digits does not mean bigger: $0.405 < 0.45 < 0.5$. Compare place by place from the left (or pad with zeros: 0.405, 0.450, 0.500).

$-5 > -2$ is FALSE. With negatives, the number that looks bigger is smaller: -5 is 3 below -2.

Read the instruction: 'starting with the largest' reverses the order. And always give the final list in the original forms, not as your decimals.

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6 The four operations: fractions, negatives and order of operations

Syllabus 1.6

Brackets first, then indices, then multiply and divide (left to right), then add and subtract (left to right). For fractions, change mixed numbers to improper fractions first; to divide, multiply by the reciprocal.

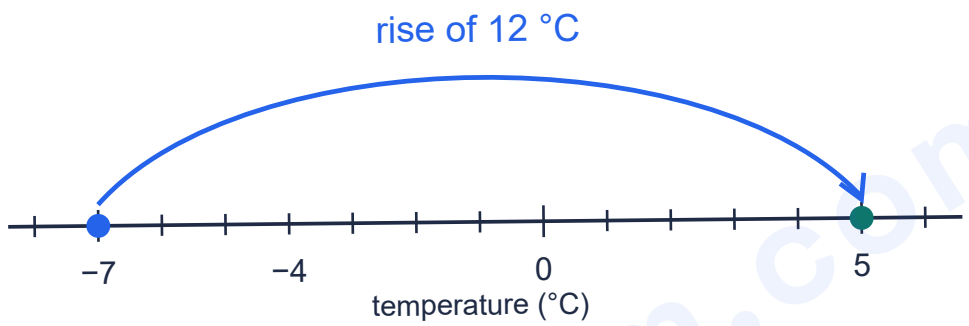

Fig. 1: from −7 °C to 5 °C is 7 degrees up to zero and 5 more, a rise of 5 − (−7) = 12 °C

Worked example: Temperature changes

(a) The temperature rises from -7 °C to 5 °C. By how much does it rise? (b) At 6 pm it is 3 °C; overnight it falls by 9 degrees. What is the new temperature?

1. (a) Rise = final − start $= 5 - (-7) = 5 + 7$ (Fig. 1).
2. (b) Fall means subtract: $3 - 9$.

Answer: (a) 12 °C (b) -6 °C

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Order, signs, then the fraction rules.

- ✓ **Order:** brackets, indices, $\times \div$ left to right, $+ -$ left to right.
- ✓ **Signs:** subtracting a negative adds; same signs multiply or divide to a positive.
- ✓ **Mixed numbers:** make them improper first.
- ✓ **Add/subtract fractions:** common denominator. **Divide:** multiply by the reciprocal.
- ✓ **Decimals:** count decimal places to multiply; scale both numbers to divide by a whole number.

Watch out!

$-3^2 = -9$ because the square is done before the minus sign, but $(-3)^2 = 9$. On a calculator, type the brackets.

$20 - 12 + 2$ is 10, not 6. Addition does not come before subtraction: work left to right.

$\frac{2}{5} + \frac{1}{3}$ is NOT $\frac{3}{8}$. Never add the bottoms: use a common denominator, $\frac{6}{15} + \frac{5}{15} = \frac{11}{15}$.

Dividing by a number less than 1 makes the answer BIGGER: $4.8 \div 0.06 = 80$, not 0.8. Check the size of your answer makes sense.

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7 Indices: the laws of indices

Syllabus 1.7

With the same base: $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$. Also $a^0 = 1$, $a^{-n} = \dfrac{1}{a^n}$ and (Extended) $a^{\frac{1}{n}} = \sqrt[n]{a}$.

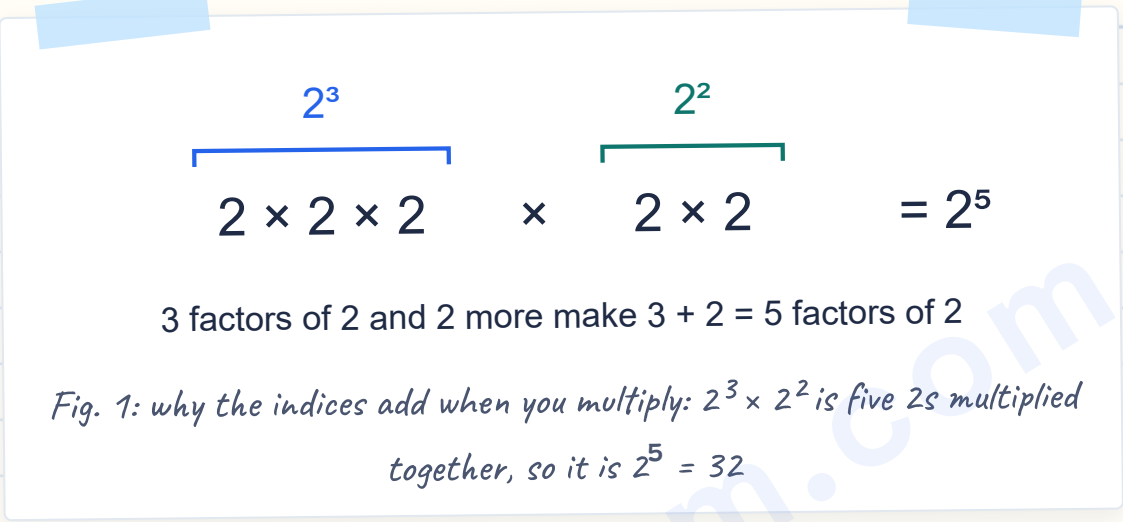

Fig. 1: why the indices add when you multiply: $2^3 \times 2^2$ is five 2s multiplied together, so it is $2^5 = 32$

Worked example: Using the laws

Find the value of (a) $2^{-3} \times 2^4$ (b) $(2^3)^2$ (c) $2^3 \div 2^4$.

1. (a) Same base, multiplying: add the indices, $-3 + 4 = 1$, so $2^1 = 2$.
2. (b) Power of a power: multiply the indices, $3 \times 2 = 6$, so $2^6 = 64$.
3. (c) Dividing: subtract the indices, $3 - 4 = -1$, so $2^{-1} = \dfrac{1}{2}$.

Answer: (a) 2 (b) 64 (c) $\dfrac{1}{2}$

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Same base first, then the laws.

- ✓ **Multiply:** add indices. **Divide:** subtract. **Power of a power:** multiply.
- ✓ **Zero:** $a^0 = 1$. **Negative:** $a^{-n} = \frac{1}{a^n}$; a fraction flips.
- ✓ **Fractional (Extended):** bottom = root, top = power; root first.
- ✓ **Equations:** write both sides as powers of the same base, then compare indices.
- ✓ **Check:** a negative index never makes the answer negative.

Watch out!

The laws only work with the SAME base, and the base never changes: $3^4 \times 3^2 = 3^6$, not 9^6. And $2^3 \times 5^2$ cannot be combined at all.

A negative index does NOT make the number negative: $7^{-2} = \frac{1}{49}$, not -49 and not -14.

$8^{-\frac{2}{3}}$ is not $8 \times -\frac{2}{3}$. An index is never multiplied by the base; it tells you to take roots, powers and reciprocals.

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8 Standard form

Syllabus 1.8

A number in standard form is $A \times 10^n$, where $1 \leq A < 10$ and n is an integer. The power n counts how many places the digits move: positive for large numbers, negative for small ones.

340 000 = 3.4 × 100 000 = 3.4 × 10^5

Fig. 1: the decimal point moves 5 places to make a number between 1 and 10, so the power is 5

Worked example: Into standard form

Write (a) $4\,060\,000$ and (b) $0.000\,089$ in standard form.

1. (a) Put the point after the first non-zero digit: 4.06.
2. Count the places from 4.06 back to $4\,060\,000$: the point moves 6 places, and the number is large, so the power is $+6$.
3. (b) The first non-zero digit is 8, so $A = 8.9$.
4. From $0.000\,089$ to 8.9 the point moves 5 places right; the number is small, so the power is -5.

Answer: (a) 4.06×10^6 (b) 8.9×10^{-5}

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Front number between 1 and 10, then the power.

- ✓ **Shape:** $A \times 10^n$ with $1 \leq A < 10$ and n an integer.
- ✓ **Large or small:** numbers of 10 or more have $n > 0$; numbers below 1 have $n < 0$.
- ✓ **Multiply/divide:** front numbers together, then add/subtract the powers.
- ✓ **Add/subtract:** match the powers first (or use ordinary numbers).
- ✓ **Tidy up:** if the front number is 10 or more (or below 1), adjust it and the power.

Watch out!

34×10^5 is a correct number but it is NOT in standard form, because 34 is not less than 10. Examiners give no mark for it. Fix it to 3.4×10^6.

With negative powers, the bigger-looking power is the smaller number: 10^{-5} is a hundred times SMALLER than 10^{-3}.

You cannot add powers when adding numbers: $2 \times 10^5 + 3 \times 10^4$ is NOT 5×10^9. Index laws are for multiplying and dividing only.

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9 Rounding and estimation

Syllabus 1.9

Find the last digit you are keeping, then look at the next digit only: 5 or more rounds up, 4 or less leaves it. Keep the size of the number with zeros (38 652 is 38 700 to 3 s.f., not 387). To estimate, round every number to 1 significant figure first.

past the halfway mark, so 5764 rounds up to 6000

Fig. 1: 5764 lies between 5000 and 6000 and is beyond the halfway value 5500, so to the nearest thousand it is 6000

Worked example: Nearest 10, 100, 1000

Write 5764 correct to (a) the nearest thousand (b) the nearest ten.

1. (a) Thousands digit is 5; the next digit is 7, which is 5 or more, so round up (Fig. 1).
2. Fill the places after it with zeros: 6000.
3. (b) Tens digit is 6; the next digit is 4, so the 6 stays: 5760.

Answer: (a) 6000 (b) 5760

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Find the last digit kept, look at the next one.

- ✓ **Deciding digit:** 5 or more rounds up; 4 or less does not.
- ✓ **Significant figures** start at the first non-zero digit.
- ✓ **Place holders:** fill with zeros in whole numbers; keep trailing zeros in decimals (8.00).
- ✓ **Estimate:** every number to 1 s.f., show the rounded numbers, then calculate.
- ✓ **Context:** round up or down so the answer makes sense; money to 2 d.p.

Watch out!

7.9963 to 2 d.p. is 8.00, not 8. The zeros show the accuracy asked for; writing 8 gives only 0 decimal places.

38 652 to 3 s.f. is 38 700, not 387. Rounding must not change the size of the number.

Dividing by 0.5 gives a bigger answer, not a smaller one: $400 \div 0.5 = 800$, not 200.

Do not round half-way through a calculation. Keep the full calculator value and round only the final answer, or your 3 s.f. answer may be wrong.

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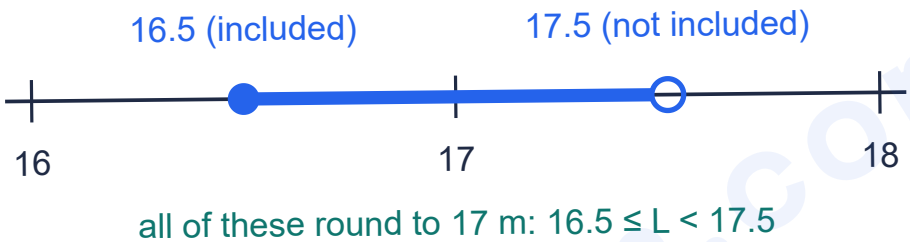
10 Limits of accuracy: upper and lower bounds

Syllabus 1.10

The bounds are half a rounding unit below and above the given value: for 17 m to the nearest metre, $16.5 \leq L < 17.5$. (Extended) For a maximum, combine the values that make the answer biggest: when dividing, that is the upper bound over the LOWER bound.



all of these round to 17 m: 16.5 ≤ L < 17.5

Fig. 1: lengths that round to 17 m to the nearest metre. The filled circle includes 16.5; the open circle excludes 17.5, which would round up to 18

Worked example: Nearest metre

A length L is 17 m, correct to the nearest metre. Write down the upper and lower bounds and the error interval.

1. The unit is 1 m, so half a unit is 0.5 m.
2. Lower bound $17 - 0.5 = 16.5$ m; upper bound $17 + 0.5 = 17.5$ m.
3. 16.5 rounds to 17, so it is included; 17.5 rounds to 18, so it is not (Fig. 1).

Answer: Lower 16.5 m, upper 17.5 m; $16.5 \leq L < 17.5$

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Unit, half unit, bounds, then combine.

- ✓ **Unit:** read it from the accuracy (nearest 5 kg, 1 d.p., 2 s.f.).
- ✓ **Bounds:** value $\pm$ half a unit.
- ✓ **Interval:** lower $\leq x <$ upper.
- ✓ **Add, multiply (Extended):** max with max, min with min.
- ✓ **Subtract, divide (Extended):** max with the OTHER value's min, and vice versa.

Watch out!

The upper bound of 17 m is 17.5 m, not 17.4 or 17.49. Cambridge writes the upper bound as 17.5, even though 17.5 itself is excluded from the interval.

60 kg to the nearest 5 kg is NOT $59.5 \leq m < 60.5$. The half unit is half of 5, not half of 1.

For the lower bound of a speed, do NOT use both lower bounds: $99.5 \div 12.35 = 8.06$ is bigger than the true minimum. Dividing by the larger time gives the smaller answer.

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11 Ratio and proportion

Syllabus 1.11

Find the value of **one part**. Divide the amount you know by the number of parts it stands for, then multiply by the number of parts you want.

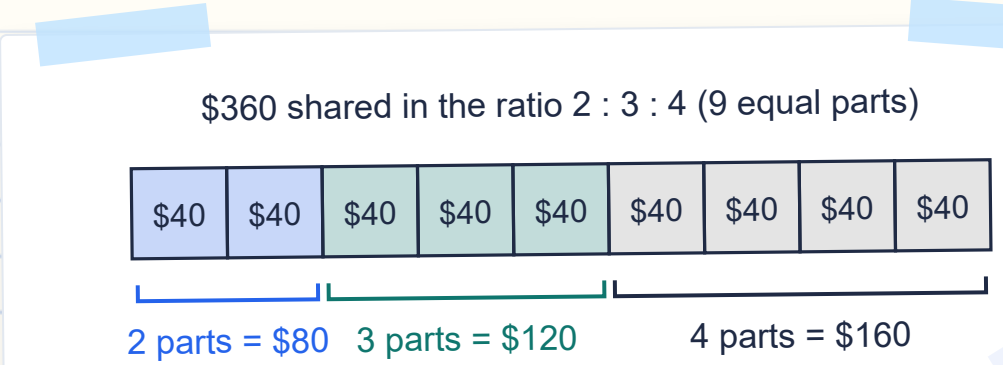

Fig. 1: a bar model for sharing $360 in the ratio 2 : 3 : 4. Every block is worth $360 ÷ 9 = $40

Worked example: Simplify with mixed units

Write $1.5\ \text{kg} : 600\ \text{g}$ in its simplest form.

1. Change to the same unit: $1.5\ \text{kg} = 1500\ \text{g}$, so the ratio is $1500 : 600$.
2. The highest common factor of 1500 and 600 is 300.
3. $1500 \div 300 = 5$ and $600 \div 300 = 2$.

Answer: $5 : 2$

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Find one part, then scale.

- ✓ **Same units** before you simplify or compare.
- ✓ **Simplify** by dividing every part by the highest common factor.
- ✓ **Share:** add the parts, divide the total by that number, multiply back.
- ✓ **Difference given:** the difference in parts matches the difference in amounts.
- ✓ **Proportion:** find the value for one unit, then multiply; ask whether more of one means more or less of the other.

Watch out!

Simplifying $1.5\text{ kg} : 600\text{ g}$ as $1.5 : 600 = 1 : 400$ is wrong: the parts were in different units. Convert first, then simplify.

To share $\$360$ in the ratio $2 : 3 : 4$, do not divide by 2, 3 or 4. Divide by the TOTAL number of parts, 9.

Map scales hide a units trap: $1\text{ km} = 100\,000\text{ cm}$, not 1000 cm. Write the conversion down before you divide.

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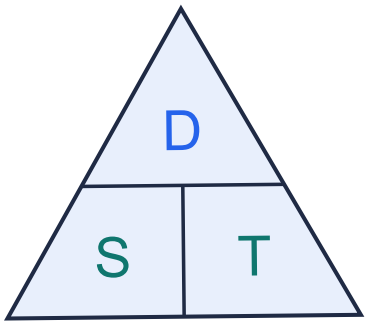
12 Rates, average speed and best value

Syllabus 1.12

A rate is one quantity **per** one unit of another. Read the units: km/h means km ÷ h, so distance = speed × time and time = distance ÷ speed. Average speed = $\frac{\text{total distance}}{\text{total time}}$.

S = D ÷ T

km/h = km ÷ h



D = S × T

T = D ÷ S

cover the quantity you want: the other two show the operation

Fig. 1: the speed, distance and time triangle. Distance (D) sits on top, so it is speed (S) times time (T)

Worked example: Average speed

A cyclist travels 45 km in 3 hours 45 minutes. What is their average speed?

1. Change the time to hours: 45 min $= 0.75$ h, so the time is 3.75 h.
2. Speed $= 45 \div 3.75$.

Answer: 12 km/h

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Write the rate with its units.

- ✓ **Units first:** km/h means km ÷ h; that tells you whether to multiply or divide.
- ✓ **Time in hours:** minutes ÷ 60, so 15 min $= 0.25$ h.
- ✓ **Average speed** $=$ total distance ÷ total time, including stops.
- ✓ **Given formulas** (density, pressure): substitute with consistent units.
- ✓ **Check** that the answer is sensible and has units.

Watch out!

3 hours 45 minutes is NOT 3.45 hours. 45 minutes is $\frac{45}{60} = 0.75$ of an hour, so the time is 3.75 hours.

Fuel consumption can be given as litres per 100 km OR km per litre. Check which before multiplying: 6.4 litres per 100 km is NOT 6.4 km per litre.

If the area is in cm^2 but the answer must be in N/m^2, convert the area first: $1\ \text{m}^2 = 10\,000\ \text{cm}^2$, not 100 cm^2.

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13 Percentages: change, reverse percentages

Syllabus 1.13

Use a **multiplier**: an increase of $r\%$ multiplies by $1 + \frac{r}{100}$, a decrease of $r\%$ by $1 - \frac{r}{100}$. To go back to the original (reverse percentage), DIVIDE by the multiplier.

Fig. 1: a double number line for a reverse percentage. The sale price $56 matches 80% of the original price, not 100%

Worked example: As a percentage

Express 42 cm as a percentage of 1.2 m.

1. Same units: $1.2\ m = 120\ cm$.
2. $\frac{42}{120} \times 100 = 35$.

Answer: 35%

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Turn the percentage into a multiplier.

- ✓ **Percentage of:** multiply by the decimal: $18\% \to 0.18$.
- ✓ **As a percentage:** $\frac{\text{part}}{\text{whole}} \times 100$, same units.
- ✓ **Increase/decrease:** multiply by $1 \pm \frac{r}{100}$.
- ✓ **Percentage change:** change ÷ ORIGINAL $\times 100$.
- ✓ **Reverse (Extended):** divide the new amount by the multiplier.

Watch out!

Divide by the WHOLE, not by the bigger number out of habit. 'What percentage of 36 is 45?' has 36 on the bottom, giving 125%, not 80%.

Percentage change uses the ORIGINAL value on the bottom. From $\$64$ to $\$52$ is $\frac{12}{64}$, not $\frac{12}{52}$.

For a reverse percentage, do NOT take 20% of the sale price and add it on: $56 + 0.2 \times 56 = 67.20$, not 70. The 20% was of the ORIGINAL price, so divide by 0.8.

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14 Simple and compound interest

Syllabus 1.13

Simple interest: $I = \dfrac{PRT}{100}$, the same every year. Compound interest: amount $= P\left(1 + \dfrac{r}{100}\right)^n$; subtract P if the question asks for the interest only.

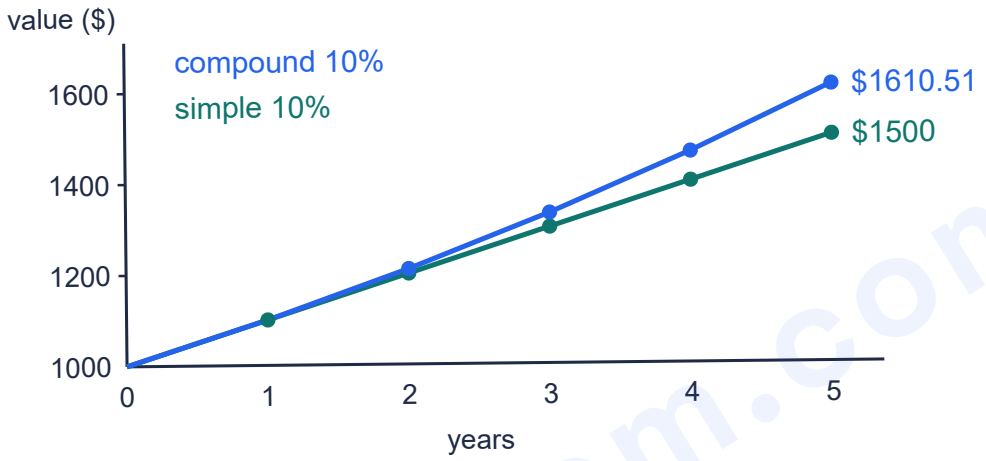

Fig. 1: $1000 invested at 10% per year. Simple interest adds $100 each year (a straight line); compound interest multiplies by 1.1 each year (a curve that pulls ahead)

Worked example: Simple interest

$2400 is invested at 3.5% per year simple interest for 4 years. Find the total value at the end.

1. Interest $= \dfrac{2400 \times 3.5 \times 4}{100} = 336$.
2. Total $= 2400 + 336$.

Answer: $2736

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Simple adds, compound multiplies.

- ✓ **Simple:** $I = \dfrac{PRT}{100}$, the same every year.
- ✓ **Compound:** amount $= P \times \left(1 + \dfrac{r}{100}\right)^n$.
- ✓ **Interest or amount?** Subtract P if only the interest is asked for.
- ✓ **Number of years:** try values of n until the target is passed.
- ✓ **Round at the end** to the nearest cent.

Watch out!

Check whether the question asks for the INTEREST or the final AMOUNT. Giving $\$336$ when the total $\$2736$ was asked for loses the mark.

Compound interest is NOT the simple interest for one year multiplied by the number of years. $2400 \times 1.035 \times 4$ is meaningless; the power goes on the multiplier: 2400×1.035^4.

Do not round the multiplier or round year by year. Keep the full calculator value and round only the final money answer to the nearest cent.

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15 Using a calculator efficiently

Syllabus 1.14

Put brackets round every top, every bottom and everything under a root. Round **only the final answer**, and translate the display: 4.8 in money means $\$4.80$, and 3.25 hours means 3 hours 15 minutes.

Worked example: A fraction

Work out $\dfrac{5.6 + 3.82}{2.4 \times 1.15}$. Give your answer to 3 significant figures.

1. Estimate first: $\dfrac{6 + 4}{2 \times 1} = 5$, so the answer is a small number, roughly 3 to 5.
2. Key in $(5.6 + 3.82) \div (2.4 \times 1.15)$.
3. Display: $3.413\,043\ldots$

Answer: 3.41 (typing $5.6 + 3.82 \div 2.4 \times 1.15$ without brackets gives 7.43, which is wrong)

Estimate, key in with brackets, round last.

- ✓ **Estimate** first with 1 s.f. values, so you can spot a keying error.
- ✓ **Brackets** round every numerator, denominator and expression under a root.
- ✓ **Times:** minutes ÷ 60 gives the decimal of an hour; decimal × 60 gives minutes.
- ✓ **Write down** the full display before rounding when asked for all the figures.
- ✓ **Money** to 2 decimal places; other answers to 3 s.f.

Watch out!

Typing $5.6 + 3.82 \div 2.4 \times 1.15$ divides only the 3.82. Without brackets the calculator does the division before the addition.

2 hours 40 minutes is NOT 2.4 hours. $38.50 \times 2.4 = \$92.40$ undercharges by more than $\$10$.

Rounding in the middle of a calculation is the commonest way to lose an accuracy mark. Keep the full value on the calculator and round once, at the end.

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16 Time: 12- and 24-hour clock, time calculations

Syllabus 1.15

To find the time between two times, **count on**: to the next whole hour, then whole hours, then the minutes left. Never subtract times like ordinary decimals, because an hour has 60 minutes, not 100.

Worked example: Converting units of time

Write 2.4 days in hours and minutes.

1. $2.4 \times 24 = 57.6$ hours.
2. The decimal part is a fraction of an hour: $0.6 \times 60 = 36$ minutes.

Answer: 57 hours 36 minutes

Count on, in base 60.

- ✓ **24-hour clock:** four digits; add 12 for p.m. times after 1 p.m.
- ✓ **Intervals:** count on to the next hour, then whole hours, then minutes.
- ✓ **Decimal hours:** decimal part × 60 gives minutes.
- ✓ **Timetables:** read down one column for one journey.
- ✓ **Time zones:** 'ahead' means add the difference; 'behind' means subtract.

Watch out!

12.40 a.m. is just after MIDNIGHT, so it is 00 40, not 12 40. And 57.6 hours is 57 hours 36 minutes, not 57 hours 60 minutes.

Do not subtract times as decimals: $13.22 - 8.47 = 4.75$, which is NOT 4 h 75 min or 4.75 hours. The correct interval is 4 h 35 min.

'Ahead' means the clock shows a LATER time. When you fly to a place that is ahead, add the time difference to the arrival time in your home time; do not subtract it.

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17 Money and currency conversion

Syllabus 1.16

If \$1 = €0.92, then dollars $\times 0.92$ gives euros and euros $\div 0.92$ gives dollars. Check the answer makes sense: one dollar buys LESS than one euro, so the number of euros is smaller.

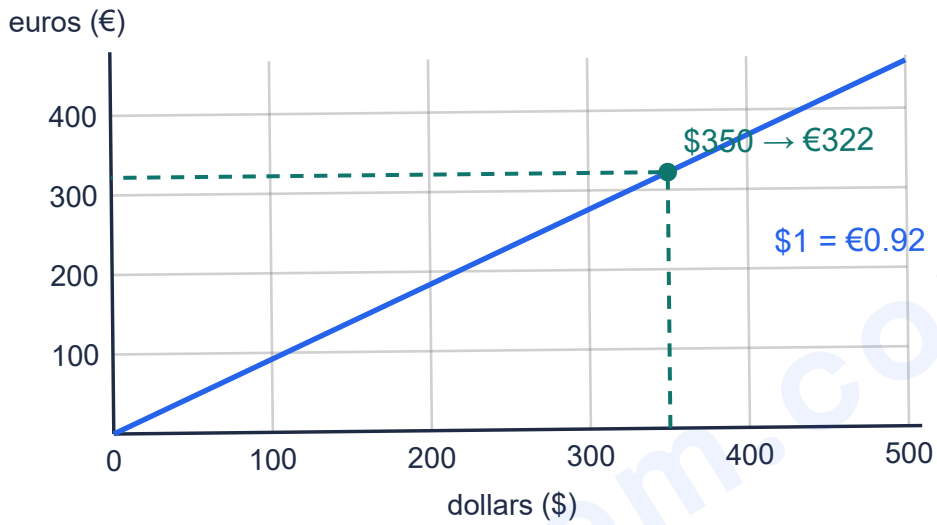

Fig. 1: a conversion graph for \$1 = €0.92. Reading up from \$350 and across gives €322

Worked example: Change from a note

Zara buys 3 notebooks at $\$2.35$ each and 2 pens at $\$0.85$ each. She pays with a $\$20$ note. How much change does she get?

1. Notebooks: $3 \times 2.35 = \$7.05$. Pens: $2 \times 0.85 = \$1.70$.
2. Total: $7.05 + 1.70 = \$8.75$.
3. Change: $20 - 8.75$.

Answer: $\$11.25$

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Same currency, two decimal places.

- ✓ **One unit** throughout: all dollars or all cents.
- ✓ **Exchange rate** $\$1 = k$ units: multiply dollars by k; divide the other currency by k.
- ✓ **Sense check:** does the answer have more or fewer units than you started with?
- ✓ **Compare** prices only after changing them into the same currency.
- ✓ **Commission:** take it off before converting, unless told otherwise.

Watch out!

Mixing dollars and cents in one calculation causes errors of 100 times. Work entirely in dollars (85 cents $= \$0.85$) or entirely in cents.

Multiplying when you should divide gives $230 \times 0.92 = \$211.60$ for part (b). Sense-check first: $1 buys only €0.92, so a euro is worth MORE than a dollar, and €230 must be MORE than 230 dollars.

Do not subtract prices in different currencies: $185 - 150 = 35$ has no meaning because dollars and pounds are different units.

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18 Exponential growth and decay

Syllabus 1.17

Value after n years $= \text{start} \times \left(1 \pm \frac{r}{100}\right)^n$. Growth uses a multiplier bigger than 1, decay a multiplier between 0 and 1. The power is the number of times the change happens.

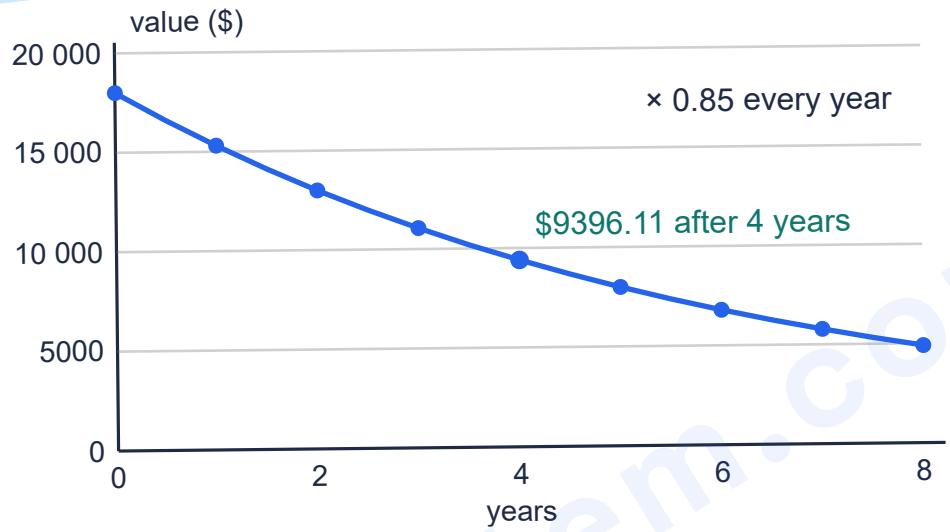

Fig. 1: a car bought for $18 000 loses 15% of its value each year, so its value is multiplied by 0.85 every year. The curve falls by a smaller amount each year but never reaches zero

Worked example: Depreciation

A car costs $\$18\,000$. Its value falls by 15% each year. Find its value after 4 years.

1. A 15% decrease leaves 85%, so the multiplier is 0.85.
2. After 4 years: $18\,000 \times 0.85^4$.
3. $= 18\,000 \times 0.522\,006\ldots = 9396.1125$

Answer: $\$9396.11$ (Fig. 1)

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Start value times multiplier to the power n.

- ✓ **Multiplier:** growth $1 + \frac{r}{100}$; decay $1 - \frac{r}{100}$.
- ✓ **After n steps:** start $\times$ multiplier^n.
- ✓ **Number of steps:** try values of n and show the values either side of the target.
- ✓ **Backwards:** divide by multiplier^n.
- ✓ **Rate:** multiplier $= \sqrt[n]{\text{end} \div \text{start}}$.

Watch out!

Exponential decay is NOT a fixed amount off each year. $18\,000 - 4 \times 0.15 \times 18\,000 = \$7200$ is wrong, because each year's 15% is taken from a smaller value.

The power is the number of times the change happens, not the number of hours. In 24 hours a 6-hour halving happens 4 times, so the power is 4, not 24.

To undo 3 years of 10% decay, do not multiply by 1.1^3. Losing 10% and gaining 10% are not opposites; divide by 0.9^3 instead.

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19 Surds: simplifying and rationalising

Syllabus 1.18

Use $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ with a the largest square factor: $\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$. To rationalise, multiply top and bottom by $\sqrt{a}$, or by the conjugate: $(\sqrt{3} - 1)(\sqrt{3} + 1) = 3 - 1 = 2$.

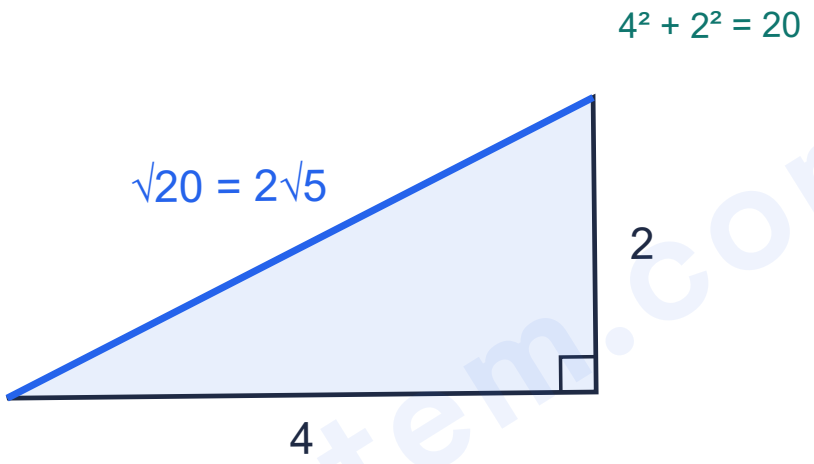

Fig. 1: a right-angled triangle with sides 4 and 2 (drawn to scale). Its hypotenuse is exactly √20 = 2√5, about 4.47

Worked example: Take out the largest square

Simplify (a) $\sqrt{72}$ (b) $\sqrt{75}$.

1. (a) The largest square factor of 72 is 36: $\sqrt{72} = \sqrt{36}\sqrt{2}$.
2. $\sqrt{36} = 6$, so $\sqrt{72} = 6\sqrt{2}$.
3. (b) $75 = 25 \times 3$, so $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$.

Answer: (a) $6\sqrt{2}$ (b) $5\sqrt{3}$

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Largest square out, surds off the bottom.

- ✓ **Simplify:** split off the largest square factor: $\sqrt{98} = 7\sqrt{2}$.
- ✓ **Collect** only like surds, after simplifying each one.
- ✓ **Expand** every term; $\sqrt{a} \times \sqrt{a} = a$.
- ✓ **Rationalise** $\dfrac{k}{\sqrt{a}}$ by multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$.
- ✓ **Conjugate:** for $p + \sqrt{q}$ on the bottom, multiply by $p - \sqrt{q}$.

Watch out!

$\sqrt{72} = 2\sqrt{18}$ is true but NOT fully simplified, because 18 still has the square factor 9. Use the LARGEST square factor, or keep going until no square factor is left.

$\sqrt{200} - \sqrt{32}$ is NOT $\sqrt{168}$. You can only subtract surds once they are the same surd, like $10\sqrt{2} - 4\sqrt{2}$.

Multiplying $\dfrac{1}{\sqrt{3} - 1}$ by $\dfrac{\sqrt{3} - 1}{\sqrt{3} - 1}$ does not help: the bottom becomes $4 - 2\sqrt{3}$, still a surd. Use the conjugate, with the opposite sign.

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