

Probability

IGCSE Maths 0580 · Topic 8 revision notes

Everything from the 5 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

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| 1. <u>Introduction to probability.</u> | 3. <u>Combined events: sample spaces and Venn diagrams</u> |
| 2. <u>Relative and expected frequencies</u> | 4. <u>Tree diagrams</u> |
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1 Introduction to probability

Syllabus 8.1

For equally likely outcomes, $P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$.
Probabilities lie from 0 (impossible) to 1 (certain), and $P(\text{not } A) = 1 - P(A)$.

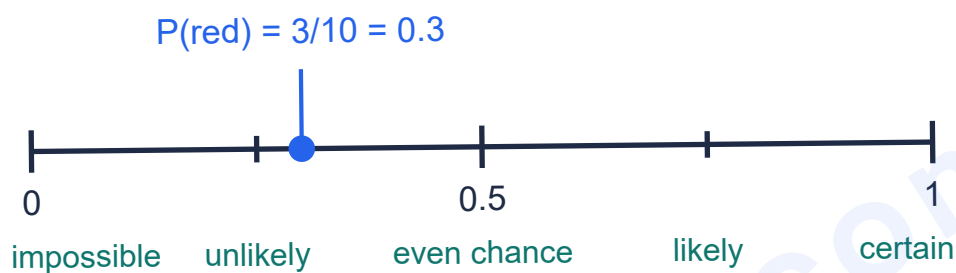


Fig. 1: the probability scale; picking a red counter from a bag of 3 red and 7 other counters has probability 0.3, so it is unlikely

Worked example: Counters in a bag

A bag contains 3 red, 5 blue and 2 green counters. One counter is taken at random. Find the probability that it is (a) red (b) not green (c) yellow.

1. Total number of counters = $3 + 5 + 2 = 10$, all equally likely.
2. (a) 3 are red: $P(\text{red}) = \frac{3}{10}$.
3. (b) $3 + 5 = 8$ are not green: $\frac{8}{10} = \frac{4}{5}$.
4. (c) There are no yellow counters, so it is impossible: 0.

Answer: (a) $\frac{3}{10}$ (b) $\frac{4}{5}$ (c) 0

Wanted outcomes over all outcomes.

- ✓ **Scale:** 0 impossible, 1 certain; nothing outside 0 to 1.
- ✓ **Equally likely:** $P = \text{number wanted} \div \text{total number}$.
- ✓ **Not happening:** $P(\text{not } A) = 1 - P(A)$ (Extended: $P(A')$).
- ✓ **All outcomes:** the probabilities add up to 1, so a missing one is 1 minus the rest.
- ✓ **Format:** fraction, decimal or percentage, never "3 out of 10".

Watch out!

A probability bigger than 1 or below 0 is always wrong. If you get one, you have probably divided the wrong way round.

The rule $1 - p$ only works for an event and its opposite. $P(\text{not blue})$ is $1 - P(\text{blue})$; it is not $P(\text{red})$, and it is not $1 - P(\text{red})$.

In a Venn diagram, "likes coffee" includes the overlap. Using only the "coffee only" region (10) is the usual slip.

2 Relative and expected frequencies

Syllabus 8.2

Relative frequency = $\frac{\text{number of times it happened}}{\text{number of trials}}$, an estimate of the probability that gets more reliable with more trials. Expected frequency = probability $\times$ number of trials.

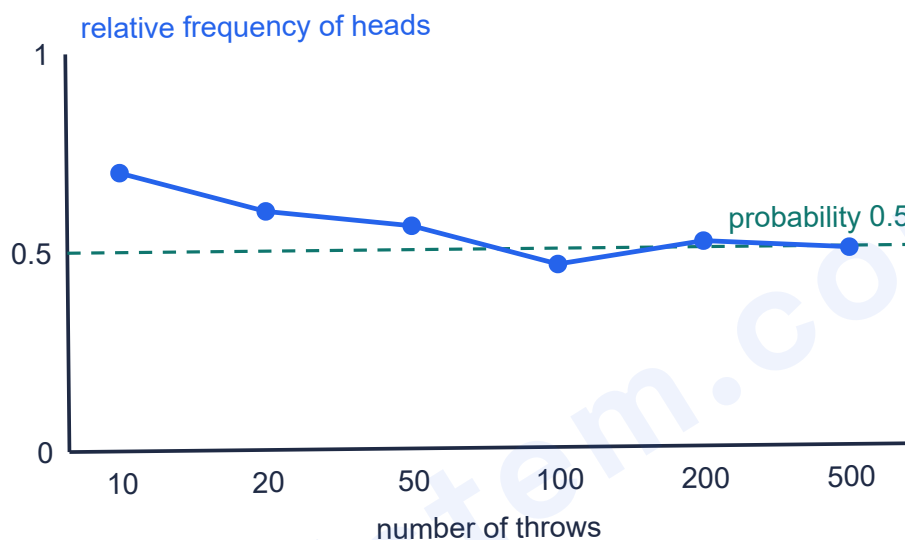


Fig. 1: relative frequency of heads after 10, 20, 50, 100, 200 and 500 throws of a coin (7, 12, 28, 46, 103 and 248 heads); it settles towards 0.5 as the number of trials grows

Worked example: A drawing pin

A drawing pin is dropped 80 times. It lands point up 37 times. Estimate the probability that it lands point up.

1. The two ways of landing are not equally likely, so we must experiment.

2. Relative frequency = $\frac{37}{80} = 0.4625$.

Answer: 0.4625 (about 0.463)

Estimate from trials, predict from probability.

- ✓ **Relative frequency:** $\text{times it happened} \div \text{number of trials}$.
- ✓ **Reliability:** more trials, better estimate.
- ✓ **Combining:** add the successes and add the trials; never average the fractions.
- ✓ **Fair or biased:** compare the relative frequency with the fair probability.
- ✓ **Expected frequency:** $\text{probability} \times \text{number of trials}$.

Watch out!

A relative frequency is an **ESTIMATE**. With only 10 throws (Fig. 1) a fair coin gave 0.7. Never decide a dice is biased from a handful of trials.

Do not average two relative frequencies: $\frac{1}{2}(0.4 + 0.275)$ gives the small group as much weight as the big one. Add the counts and the trials instead.

Expected frequency is probability **TIMES** trials, not trials divided by the probability. $240 \div 0.35 = 686$ is bigger than the number of spins, which is impossible.

3 Combined events: sample spaces and Venn diagrams

Syllabus 8.3

In a sample space diagram every cell is equally likely, so $P = \frac{\text{cells in the event}}{\text{total cells}}$. In a Venn diagram, fill the overlap first, then the rest of each circle, then the outside; the probability is the region's count over the total.

		first dice					
		1	2	3	4	5	6
second dice	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$$P(\text{total } 8) = \frac{5}{36}$$

Fig. 1: sample space for the total of two fair dice; 5 of the 36 equally likely cells give a total of 8

Worked example: Two dice

Two fair dice are rolled and the scores are added. Use Fig. 1 to find (a) $P(\text{total } 8)$ (b) $P(\text{total } 10 \text{ or more})$ (c) $P(\text{a double})$.

1. There are $6 \times 6 = 36$ equally likely cells.

2. (a) 8 appears 5 times: $\frac{5}{36}$.

3. (b) 10 appears 3 times, 11 twice and 12 once: $\frac{6}{36} = \frac{1}{6}$.

4. (c) Doubles lie on the diagonal (1,1) to (6,6): $\frac{6}{36} = \frac{1}{6}$.

Answer: (a) $\frac{5}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{6}$

Draw it, count it, divide.

- ✓ **Sample space:** a grid of every pair of outcomes; each cell equally likely.
- ✓ **Order matters:** (2, 6) and (6, 2) are separate cells.
- ✓ **Venn:** overlap first, then each circle minus the overlap, then the outside.
- ✓ **Check:** all regions add up to the total.
- ✓ **Notation (Extended):** $\cap$ and, $\cup$ or, ' not.

Watch out!

(2, 6) and (6, 2) are DIFFERENT outcomes: the first dice shows 2 in one and 6 in the other.

Counting them once is why students get $\frac{3}{36}$ instead of $\frac{5}{36}$ for a total of 8.

Do not put 17 in the football-only region: 17 is the WHOLE football circle, which already includes the 5 who play both.

$P(A \cup B)$ is NOT $P(A) + P(B)$ when the sets overlap: adding counts the overlap twice. Use the regions of the diagram.

4 Tree diagrams

Syllabus 8.3

Probabilities go beside the branches and outcomes at the ends. Multiply ALONG a path for 'this and then that'; ADD the results of different paths for 'this or that'. The branches from any one point add up to 1.

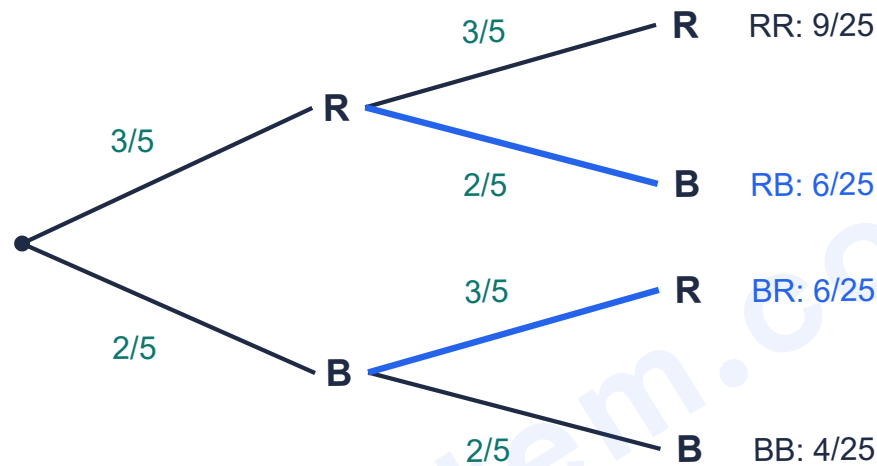


Fig. 1: two counters taken with replacement from a bag of 3 red and 2 blue; the highlighted paths give one of each colour, $\frac{6}{25} + \frac{6}{25} = \frac{12}{25}$

Worked example: With replacement

A bag holds 3 red and 2 blue counters. A counter is taken, its colour noted, and it is put back. A second counter is then taken. Find (a) $P(\text{both red})$ (b) $P(\text{one of each colour})$.

1. Put back, so the second pick has the same probabilities: $\frac{3}{5}$ red, $\frac{2}{5}$ blue (Fig. 1).
2. (a) Multiply along R then R: $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$.
3. (b) Two paths: RB gives $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$, and BR gives $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$.
4. Add the paths: $\frac{6}{25} + \frac{6}{25} = \frac{12}{25}$.

Answer: (a) $\frac{9}{25}$ (b) $\frac{12}{25}$

Multiply along, add between paths.

- ✓ **Draw:** probabilities beside the branches, outcomes at the ends.
- ✓ **Check:** each set of branches adds to 1.
- ✓ **And:** multiply along a path.
- ✓ **Or:** add the paths that give the event (both orders!).
- ✓ **At least one:** 1 minus $P(\text{none})$.
- ✓ **Without replacement (Extended):** reduce the totals on the second branches.

Watch out!

Forgetting one of the two orders. 'One red and one blue' can happen as RB OR as BR, so there are two paths to add.

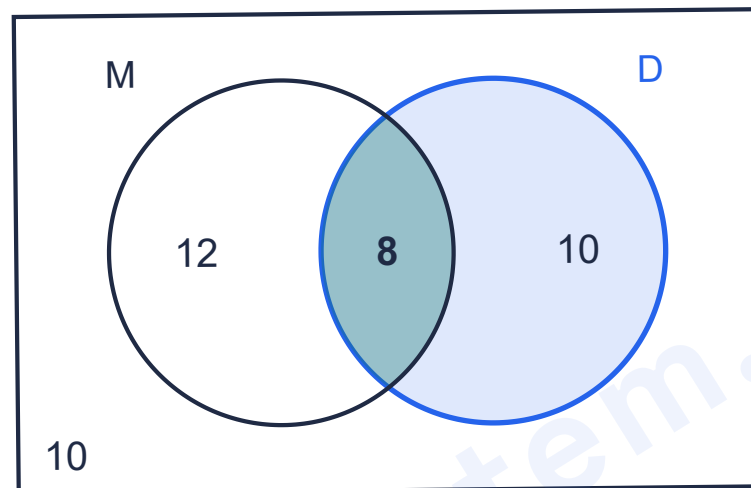
Adding along a path instead of multiplying gives nonsense: $0.8 + 0.6 = 1.4$ is bigger than 1. Along a path, always multiply.

Without replacement, BOTH the numerator and the denominator can change. After a green is taken, green becomes $\frac{2}{7}$ but red becomes $\frac{5}{7}$, not $\frac{5}{8}$.

5 Conditional probability

Syllabus 8.4

'Given that B' means look *ONLY* at the outcomes in B. Then $P(A \text{ given } B) = \frac{\text{number in both A and B}}{\text{number in B}}$. The denominator is the size of the given group, not the whole total.



Given D:
look only
inside D

$$P(M) = \frac{8}{18} \\ = \frac{4}{9}$$

Fig. 1: 40 students; M = studies music, D = studies drama. Given that a student studies drama, only the 18 in circle D count, and 8 of them study music

Worked example: From a Venn diagram

Use Fig. 1. A student is chosen at random. Find (a) the probability that the student studies music, given that they study drama (b) the probability that the student studies drama, given that they study music.

1. (a) Given drama: the group is circle D , $8 + 10 = 18$ students.
2. Of these, 8 also study music: $\frac{8}{18} = \frac{4}{9}$.
3. (b) Given music: the group is circle M , $12 + 8 = 20$ students.
4. 8 of them study drama: $\frac{8}{20} = \frac{2}{5}$.

Answer: (a) $\frac{4}{9}$ (b) $\frac{2}{5}$

The given group is the new total.

- ✓ **Spot the condition:** 'given that', 'if', 'of those who'.
- ✓ **Restrict:** keep only the outcomes in the given group.
- ✓ **Divide:** number in both $\div$ number in the given group.
- ✓ **Trees:** second branches are already conditional.
- ✓ **Reverse a tree:** wanted path $\div$ sum of all paths giving the condition.

Watch out!

'P(A given B)' and 'P(B given A)' are different. In Fig. 1 they are $\frac{4}{9}$ and $\frac{2}{5}$. Read which group is GIVEN: that group is the denominator.

Do not divide by 36 once a condition is given. 'Given that the total is 8' means the denominator is 5, not 36.

The branch value 0.4 is P(late GIVEN rain), not P(rain GIVEN late). Reversing the condition needs the division by P(late).