

Transformations and vectors

IGCSE Maths 0580 · Topic 7 revision notes

Everything from the 5 lessons of this topic on a few pages: the key idea of each lesson, the diagram to remember, the points to learn and the traps examiners set. Read it, cover it, test yourself.

1. Reflection and rotation

2. Enlargement and translation

3. Vectors in two dimensions

4. Magnitude of a vector

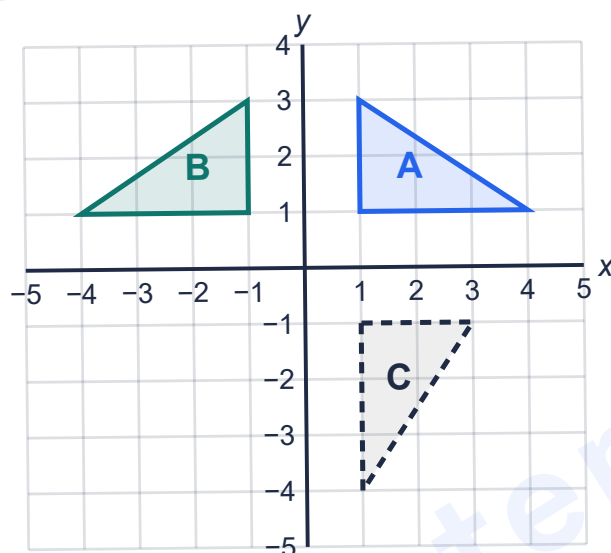
5. Vector geometry

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1 Reflection and rotation

Syllabus 7.1

To describe a reflection fully, give the equation of the mirror line. To describe a rotation fully, give the angle, the direction (clockwise or anticlockwise) and the centre. Always describe ONE single transformation.



$A \rightarrow B$:
reflection in
the y-axis ($x = 0$)

$A \rightarrow C$:
rotation 90°
clockwise about
the origin $(0, 0)$

Fig. 1: triangle A is reflected in the y-axis to give B, and rotated 90° degrees clockwise about the origin to give C; all three are congruent

Worked example: Reflecting in a vertical line

Triangle P has vertices $(1, 2)$, $(3, 2)$ and $(3, 5)$. Reflect it in the line $x = 4$.

1. $x = 4$ is vertical, so every y-coordinate stays the same.
2. $(1, 2)$ is 3 units left of the mirror, so its image is 3 units right: $(7, 2)$.
3. $(3, 2)$ and $(3, 5)$ are 1 unit left, so their images are 1 unit right: $(5, 2)$ and $(5, 5)$.

Answer: $(7, 2)$, $(5, 2)$, $(5, 5)$

Flipped or turned? Then give every fact.

- ✓ **Reflection:** same perpendicular distance from the mirror, on the other side.
- ✓ **Mirror lines:** $x = k$ vertical, $y = k$ horizontal, (Extended) $y = x$ and $y = -x$.
- ✓ **Rotation:** use tracing paper, pinned at the centre.
- ✓ **Describe a reflection:** 'Reflection in the line ...' with its equation.
- ✓ **Describe a rotation:** angle, direction (unless 180°) and centre.
- ✓ **One transformation only** when the question says 'single'.

Watch out!

Mixing up $x = k$ and $y = k$. The line $x = 4$ is VERTICAL, so reflecting in it changes the x -coordinates, not the y -coordinates. Sketch the mirror line before you reflect anything.

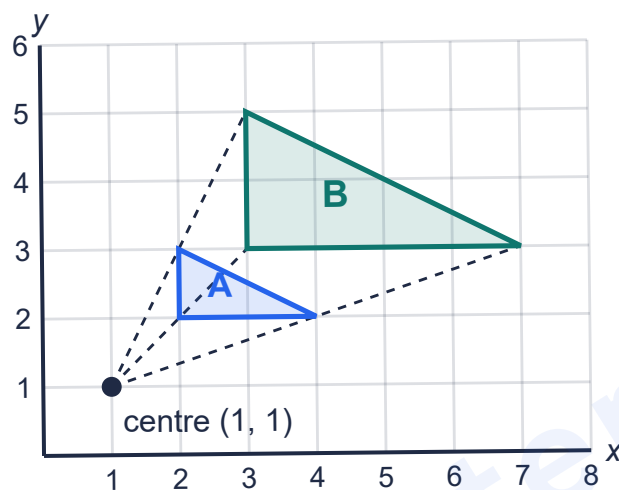
Turning the wrong way. Clockwise is the way clock hands move. A 90° clockwise turn is the same as a 270° anticlockwise turn, not a 90° anticlockwise one.

Naming two transformations, such as 'a reflection and then a translation', or leaving out the centre or direction. The question asks for a SINGLE transformation, and a rotation without its centre scores very little.

2 Enlargement and translation

Syllabus 7.1

Enlargement: every distance from the centre is multiplied by the scale factor k ; describe it with the scale factor and the centre. Translation: every point moves by the same column vector $\begin{pmatrix} x \\ y \end{pmatrix}$ (x across, y up); describe it with that vector.



scale factor 2:
every distance
from the centre
is doubled

A (2,2) (4,2) (2,3)

B (3,3) (7,3) (3,5)

Fig. 1: triangle A is enlarged by scale factor 2 from centre (1, 1); each vertex of B is twice as far from the centre, along the same ray

Worked example: Scale factor 2 from a centre

Triangle A has vertices (2, 2), (4, 2), (2, 3). Enlarge it by scale factor 2, centre (1, 1).

1. Steps from the centre: (2, 2) is 1 right, 1 up; (4, 2) is 3 right, 1 up; (2, 3) is 1 right, 2 up.
2. Double each step: 2 right, 2 up; 6 right, 2 up; 2 right, 4 up.
3. Add to the centre (1, 1): (3, 3), (7, 3), (3, 5).

Answer: (3, 3), (7, 3), (3, 5): triangle B in Fig. 1

Measure from the centre; move by the vector.

- ✓ **Enlarge:** step from the centre to each vertex, multiply by k , add back to the centre.
- ✓ **Scale factor:** image length $\div$ object length.
- ✓ **Centre:** where lines through corresponding vertices meet.
- ✓ **Translate:** add the top number to x and the bottom number to y .
- ✓ **Describe:** enlargement with scale factor and centre; translation with a column vector.

Watch out!

Multiplying the coordinates when the centre is NOT the origin.

Enlarging $(2, 2)$ by scale factor 2 from $(1, 1)$ gives $(3, 3)$, not $(4, 4)$. Always measure from the centre.

Writing the scale factor as 3 (or -3) when the shape gets smaller.

Divide IMAGE length by OBJECT length: going from 3 to 1 gives $\frac{1}{3}$.

A negative scale factor means the image is on the other side of the centre, not that it is smaller.

Subtracting the wrong way round. The vector from object to image is image minus object: from $(1, 2)$

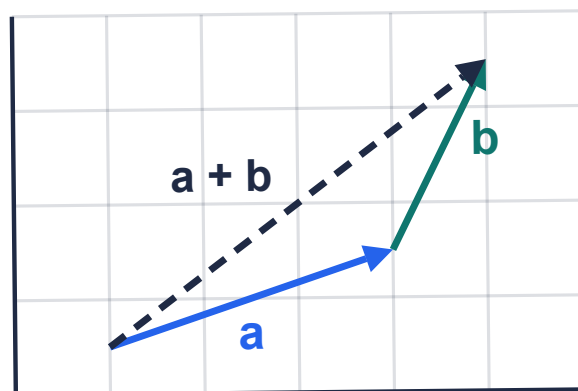
to $(5, -1)$ is $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$, and

$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ would slide the shape back again.

3 Vectors in two dimensions

Syllabus 7.2

A column vector $\begin{pmatrix} x \\ y \end{pmatrix}$ means x across and y up. Add or subtract vectors by combining the top numbers and the bottom numbers separately; $k\mathbf{a}$ multiplies both numbers by k and is parallel to $\mathbf{a}$.



$$\mathbf{a} = (3, 1)$$

$$\mathbf{b} = (1, 2)$$

$$\mathbf{a} + \mathbf{b} = (4, 3)$$

tip to tail:

follow a, then b

Fig. 1: following $\mathbf{a} = (3, 1)$ and then $\mathbf{b} = (1, 2)$ tip to tail gives the single journey $\mathbf{a} + \mathbf{b} = (4, 3)$

Worked example: Vector between two points

A is $(1, 4)$ and B is $(5, 2)$. Write $\overrightarrow{AB}$ and $\overrightarrow{BA}$ as column vectors.

1. Across: $5 - 1 = 4$. Up: $2 - 4 = -2$.

2. $\overrightarrow{BA}$ is the same journey backwards, so both signs change.

Answer: $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Top with top, bottom with bottom.

- ✓ **Between points:** $\overrightarrow{AB} = \text{end minus start}$.
- ✓ **Add or subtract:** combine the top numbers, then the bottom numbers.
- ✓ **Scalar:** $k\mathbf{a}$ multiplies both numbers by k .
- ✓ **Parallel:** one vector is a number times the other.
- ✓ **Unknowns:** make one equation from the tops and one from the bottoms.

Watch out!

Writing 'start minus end'. $\overrightarrow{AB}$ is $B - A$: the journey FROM A TO B. Getting it backwards flips both signs and gives $\overrightarrow{BA}$.

Sign slips when subtracting a negative: $5 - (-3) = 8$, not 2. Write the subtraction out in full for each number before you simplify.

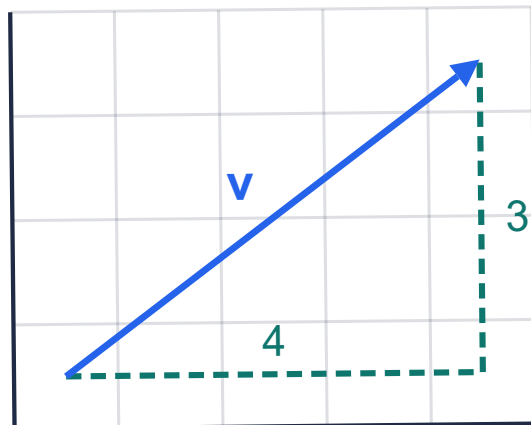
Multiplying only the top number.

$3 \begin{pmatrix} 2 \\ k \end{pmatrix}$ is $\begin{pmatrix} 6 \\ 3k \end{pmatrix}$: the scalar multiplies every component.

4 Magnitude of a vector

Syllabus 7.3

The magnitude of $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2 + y^2}$. It is written $|\mathbf{a}|$ or $|\overrightarrow{AB}|$, it is always positive, and $|\mathbf{a} + \mathbf{b}|$ is usually NOT $|\mathbf{a}| + |\mathbf{b}|$.



$$\mathbf{v} = (4, 3)$$

$$\begin{aligned} |\mathbf{v}| &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Fig. 1: $\mathbf{v} = (4, 3)$ is the hypotenuse of a right-angled triangle with sides 4 and 3, so $|\mathbf{v}| = 5$

Worked example: A whole-number magnitude

Find $|\mathbf{a}|$ when $\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

1. $|\mathbf{a}| = \sqrt{3^2 + (-4)^2}$.
2. $= \sqrt{9 + 16} = \sqrt{25}$.

Answer: $|\mathbf{a}| = 5$

Find the column vector, then Pythagoras.

- ✓ **Column vector first:** for $\overrightarrow{AB}$ or $2\mathbf{a} - \mathbf{b}$, work out the single column vector.
- ✓ **Magnitude:** $\sqrt{x^2 + y^2}$, squaring negatives with brackets.
- ✓ **Exact or rounded:** a surd such as $\sqrt{50} = 5\sqrt{2}$, or 3 significant figures.
- ✓ **Unknowns:** set $x^2 + y^2$ equal to the magnitude squared; remember $\pm$.
- ✓ **In context:** the magnitude of a velocity is a speed.

Watch out!

Squaring a negative number on a calculator without brackets: typing -4^2 gives -16 . Type $(-4)^2$, or simply square the number without its sign, since the square is always positive.

Adding magnitudes. $|\mathbf{a} + \mathbf{b}|$ is the length of ONE vector; find the column vector $\mathbf{a} + \mathbf{b}$ first, then its length. $|\mathbf{a}| + |\mathbf{b}|$ is a different number.

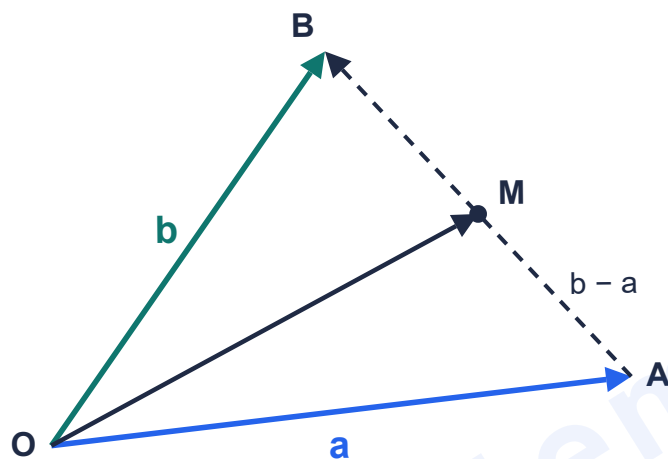
Forgetting the negative root.

$k^2 = 25$ has two solutions, 5 and -5 : both $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$ have length 13.

5 Vector geometry

Syllabus 7.4

To find any vector, follow a route of known vectors: $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$. Two vectors are parallel when one is a number times the other; three points are collinear when two such vectors are parallel AND share a point.



$$AB = b - a$$

$$AM = \frac{1}{2}(b - a)$$

$$OM = a + \frac{1}{2}(b - a) \\ = \frac{1}{2}a + \frac{1}{2}b$$

Fig. 1: with $OA = a$ and $OB = b$, the route from A to B is $b - a$, and the midpoint M of AB has position vector $(a + b)/2$

Worked example: From coordinates

A is $(2, 5)$ and B is $(7, -1)$. Find $\overrightarrow{AB}$ and the position vector of the midpoint M of AB.

1. $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.

2. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$.

3. $\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2} \begin{pmatrix} 9 \\ 4 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 2 \end{pmatrix}$.

Answer: $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$; M has position vector $\begin{pmatrix} 4.5 \\ 2 \end{pmatrix}$, so M is $(4.5, 2)$.

Follow a route of known vectors.

- ✓ **Label:** mark **a** and **b** with arrows on the diagram.
- ✓ **Route:** go along known lines; backwards along a vector means minus it.
- ✓ **Ratio:** $m : n$ along a line means $\frac{m}{m+n}$ of that line's vector.
- ✓ **Simplify:** collect the **a** terms and the **b** terms.
- ✓ **Parallel:** show one vector is a multiple of the other.
- ✓ **Collinear:** parallel AND a common point; say so in words.

Watch out!

Writing $\overrightarrow{AB} = \mathbf{a} - \mathbf{b}$. The route from *A* to *B* goes BACK along **a** (that is $-\mathbf{a}$) and then out along **b**: $-\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$.

Turning a ratio straight into a fraction. $AP : PB = 1 : 3$ means AP is $\frac{1}{4}$ of AB , not $\frac{1}{3}$; add the parts of the ratio to find the denominator.

Stopping at 'parallel'. Two parallel vectors could lie on two different lines. For collinear you must also say that the vectors share a common point.