

Capacitance

A-Level Physics 9702 · Topic 19 revision notes · A2

Charge stored per volt, capacitors in series and parallel, the energy stored, and exponential discharge through a resistor with time constant RC .

8 definitions to learn word for word, 13 equations in their own column, 10 diagrams and graphs. Read it, cover it, test yourself.

1. 19.1a Capacitors and capacitance
2. 19.1b Capacitors in series and parallel
3. 19.2 Energy stored in a capacitor
4. 19.3a Discharging a capacitor
5. 19.3b The time constant and log graphs

■ Definitions (mark-scheme wording) ■ Equations ■ Exam traps

19.1a Capacitors and capacitance

Definitions

Capacitance: the charge stored per unit potential difference ($C = Q/V$).

Farad: the capacitance of a conductor that stores one coulomb of charge per volt ($1 \text{ F} = 1 \text{ C V}^{-1}$).

Capacitance of an isolated spherical conductor: the charge on the sphere divided by its potential (relative to zero at infinity).

Parallel-plate capacitor: two conducting plates separated by an insulator (dielectric), carrying equal and opposite charges $+Q$ and $-Q$.

Equations

$$C = Q / V$$

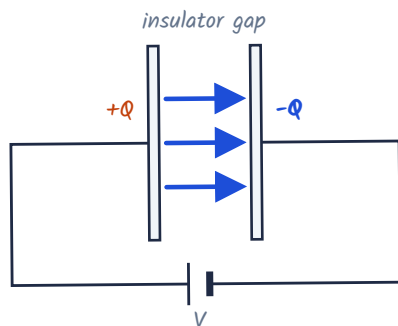
capacitance = charge on one plate / p.d.
F

$$C = 4\pi\epsilon_0 r$$

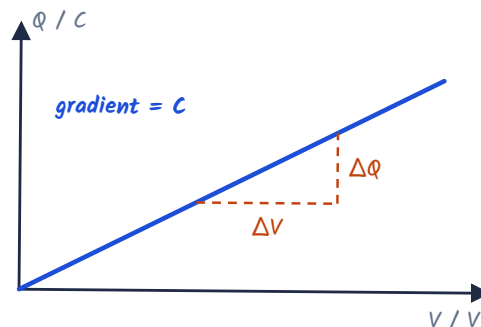
isolated conducting sphere of radius r
F

Key points

- ✓ A capacitor stores **charge** ($+Q$ on one plate, $-Q$ on the other) and hence **energy**; its net charge is zero.
- ✓ Q is the charge on **one plate**, not the total of both.
- ✓ $Q \propto V$ for a capacitor, so a Q - V graph is a straight line through the origin with **gradient C** .
- ✓ Isolated sphere: $V = Q/(4\pi\epsilon_0 r)$, so $C = 4\pi\epsilon_0 r$.
- ✓ Practical values are small: μF (10^{-6}), nF (10^{-9}), pF (10^{-12}).



Parallel-plate capacitor charged by a cell: equal and opposite charges, a uniform field between the plates.



Q against V for a capacitor: a straight line through the origin whose gradient is C .

Watch out!

- ! Use the charge on **one plate** in $C = Q/V$; the total charge on a capacitor is zero.
- ! Convert μF to F ($\times 10^{-6}$) before calculating.

19.16 Capacitors in series and parallel

Definitions

Combined capacitance: the capacitance of the single capacitor that would store the same charge for the same p.d. as the combination.

Equations

$$C = C_1 + C_2 + \dots$$

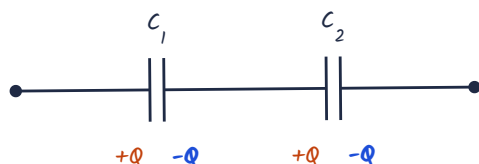
capacitors in parallel
 F

$$1/C = 1/C_1 + 1/C_2 + \dots$$

capacitors in series
 F^{-1}

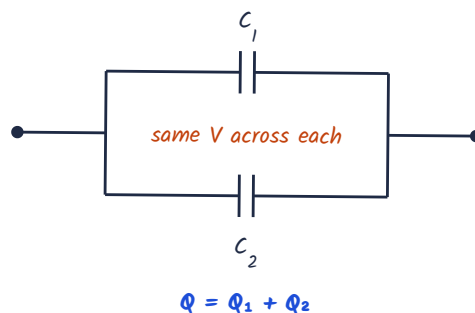
Key points

- ✓ Parallel: the **same p.d.** across each; charges add, $Q = Q_1 + Q_2$, so $CV = C_1V + C_2V$.
- ✓ Series: the **same charge Q** on each capacitor; p.d.s add, $V = V_1 + V_2$, so $Q/C = Q/C_1 + Q/C_2$.
- ✓ Parallel total is **larger** than any single capacitor; series total is **smaller** than the smallest.
- ✓ The rules are the **opposite** of those for resistors.
- ✓ Two equal capacitors: parallel gives $2C$; series gives $C/2$.



$$\text{same } Q; V = V_1 + V_2$$

Series: each capacitor carries the same charge Q ; the p.d.s add.



Parallel: each capacitor has the same p.d.; the charges add.

Watch out!

- ! Do not use the resistor rules: capacitors **add in parallel**, reciprocals add in series.
- ! After finding $1/C$ for series, remember to **take the reciprocal**.

19.2 Energy stored in a capacitor

Definitions

Energy stored in a capacitor: the electric potential energy stored, equal to the area under the potential-charge graph.

Key points

- ✓ Adding a small charge ΔQ at p.d. V needs work $\Delta W = V\Delta Q$: a thin strip under the V - Q graph.
- ✓ $V \propto Q$, so the total area is a **triangle**: $W = \frac{1}{2}QV$.
- ✓ Substituting $Q = CV$ gives $W = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C$.
- ✓ Doubling V **quadruples** the energy stored ($W \propto V^2$).
- ✓ Charging from a battery: the battery supplies QV , the capacitor stores $\frac{1}{2}QV$; the other half is **dissipated** in the resistance.

Equations

$$W = \frac{1}{2}QV$$

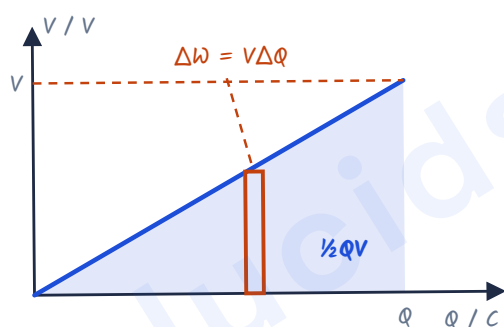
energy stored = area under V - Q graph
J

$$W = \frac{1}{2}CV^2$$

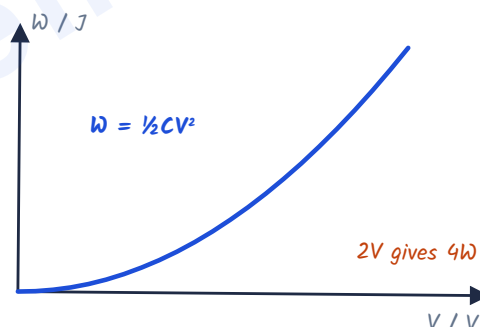
using $Q = CV$
J

$$W = \frac{1}{2}Q^2 / C$$

using $V = Q/C$
J



Energy stored = shaded triangle under the V - Q line; each thin strip is $V\Delta Q$.



Energy stored against p.d.: a parabola, $W = \frac{1}{2}CV^2$.

Watch out!

- ! Energy stored is $\frac{1}{2}QV$, not QV : the p.d. rises from zero as the charge builds up.
- ! In $W = \frac{1}{2}CV^2$, square only the V .

19.3a Discharging a capacitor

Definitions

Exponential decay: a decrease in which the quantity falls by the same fraction in equal intervals of time.

Key points

- ✓ Discharging through a resistor, Q , V and I all decay exponentially with the same time constant.
- ✓ The current is greatest at the start ($I_0 = V_0/R$) because the p.d. is greatest then.
- ✓ Rate of discharge $\propto$ charge remaining: as Q falls, V and hence I fall, so discharge slows.
- ✓ Discharge current flows the **opposite way** to the charging current.
- ✓ Area under an I - t graph = **charge** that flowed.
- ✓ Gradient of a Q - t graph = current at that instant.

Equations

$$Q = Q_0 e^{-t/RC}$$

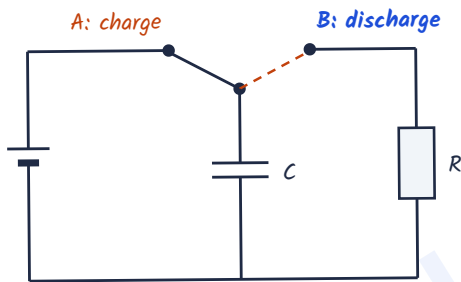
charge remaining
 C

$$V = V_0 e^{-t/RC}$$

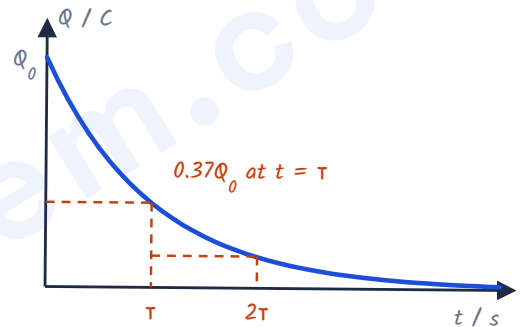
p.d. across the capacitor
 V

$$I = I_0 e^{-t/RC}$$

discharge current
 A



Charge (A) and discharge (B) circuit: switch to A to charge from the cell, then to B to discharge through R .



Discharge curve: after one time constant $\tau = RC$, Q (or V or I) has fallen to 0.37 of its initial value.

Watch out!

- ! The decay never reaches zero: quote times as multiples of τ , not a "time to discharge fully".
- ! Each time constant removes the same **fraction** (63%), not the same amount.

19.3b The time constant and log graphs

Definitions

Time constant (τ): the time taken for the charge (or p.d. or current) to fall to $1/e$ (about 0.37) of its initial value; $\tau = RC$.

Key points

- ✓ $\tau = RC$ has units of **seconds** ($\Omega \times F = s$).
- ✓ Larger R or larger C gives **slower** discharge.
- ✓ After about **5RC** the capacitor is effectively fully discharged (under 1% remains).
- ✓ Taking logs: $\ln V = \ln V_0 - t/RC$, so **$\ln V$ against t is a straight line** of gradient $-1/RC$.

Equations

$$\tau = RC$$

time constant
 s

$$\ln V = \ln V_0 - t/RC$$

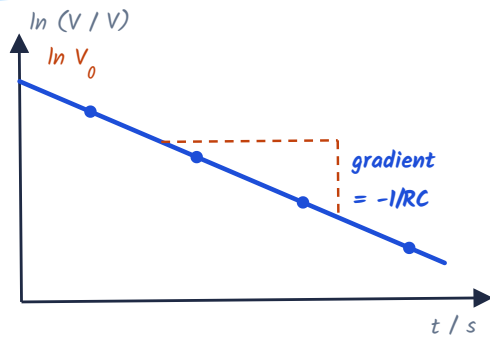
straight-line form of the decay
(no unit)

$$t_{1/2} = RC \ln 2 \approx 0.69RC$$

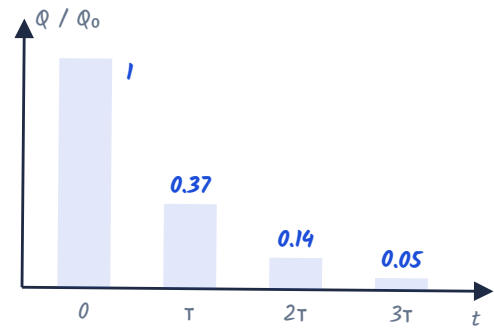
time for Q , V or I to halve
 s

✓ Time to halve: $t_{1/2} = RC \ln 2 \approx 0.69RC$.

✓ Find RC from a V - t graph: read the time for V to fall to $0.37V_0$.



$\ln V$ against t is a straight line: intercept $\ln V_0$, gradient $-1/RC$.



Each interval of τ multiplies the remaining charge by 0.37 ; each interval of $0.69RC$ halves it.

Watch out!

- ! Use natural logs ($\ln$), not $\log_{10}$, when the gradient is to give $-1/RC$.
- ! Convert μF and $k\Omega$ before multiplying: $100 \mu F \times 10 k\Omega = 1 s$.