

Motion in a circle

A-Level Physics 9702 · Topic 12 revision notes · A2

Radians and angular speed, then the centripetal acceleration and force that turn velocity without changing its size, applied to real examples.

7 definitions to learn word for word, 13 equations in their own column, 6 diagrams and graphs. Read it, cover it, test yourself.

1. 12.1a Radians and angular displacement
2. 12.1b Angular speed
3. 12.2a Centripetal acceleration and centripetal force
4. 12.2b Applying circular motion

Definitions (mark-scheme wording) Equations Exam traps

12.1a Radians and angular displacement

Definitions

Radian: the angle subtended at the centre of a circle by an arc equal in length to the radius.

Angular displacement (θ): the angle through which an object has turned about the centre of the circle, measured in radians.

Key points

- ✓ A full circle is $2\pi \text{ rad} = 360^\circ$; so $1 \text{ rad} \approx 57.3^\circ$.
- ✓ Degrees to radians: multiply by $\pi/180$.
- ✓ Arc length $s = r\theta$ only works with θ in **radians**.
- ✓ Small angles (θ in rad): $\sin \theta \approx \tan \theta \approx \theta$.

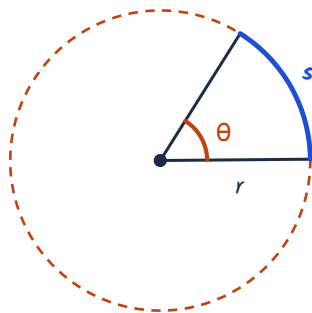
Equations

$$\theta = s / r$$

angle in radians = arc length / radius
rad

$$2\pi \text{ rad} = 360^\circ$$

one full revolution



$$\theta = s / r$$
$$s = r \cdot 1 \text{ rad}$$

Angle in radians = arc length / radius. When $s = r$, $\theta = 1 \text{ rad}$.

Watch out!

! Calculator in the wrong mode: set it to **radians** for any angle from $\theta = s/r$ or ωt .

12.16 Angular speed

Definitions

Angular speed (ω): the rate of change of angular displacement (angle turned per unit time).

Period (T): the time taken for one complete revolution.

Frequency (f): the number of revolutions per unit time.

Key points

- ✓ Every point on a rigid rotating body has the **same ω** .
- ✓ Linear (tangential) speed **increases with radius**: $v = r\omega$.
- ✓ Uniform circular motion: constant speed, constant ω , but **changing velocity** (direction changes).
- ✓ Convert rev min^{-1} to rad s^{-1} : multiply by 2π , divide by 60.

Equations

$$\omega = \Delta\theta / \Delta t$$

angular speed = angle turned / time
 rad s^{-1}

$$\omega = 2\pi / T = 2\pi f$$

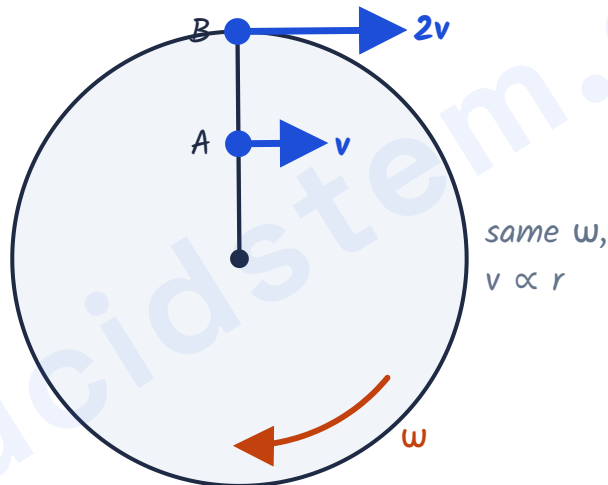
angular speed from period or frequency
 rad s^{-1}

$$v = r\omega$$

linear speed = radius $\times$ angular speed
 m s^{-1}

$$v = 2\pi r / T$$

circumference / period
 m s^{-1}



A rotating disc: points A and B turn through the same angle in the same time (same ω), but B at larger radius moves faster ($v = r\omega$).

Watch out!

- ! ω must be in rad s^{-1} , not rev s^{-1} or degrees per second.
- ! Radius r is measured from the **centre** of the circle, not the diameter.

12.2a Centripetal acceleration and centripetal force

Definitions

Centripetal acceleration: the acceleration of an object moving in a circle, directed towards the centre of the circle.

Centripetal force: the resultant force on an object moving in a circle, directed towards the centre of the circle.

Key points

- ✓ At constant speed the **velocity changes direction**, so the object accelerates (Newton's first law needs a resultant force).
- ✓ Velocity is **tangential**; force and acceleration point **along the radius to the centre**.
- ✓ Force is **perpendicular to velocity**, so it does no work: kinetic energy and speed stay constant.
- ✓ Centripetal force is not an extra force: it is provided by tension, gravity, friction, normal contact or a magnetic force.
- ✓ If the force is removed, the object moves off along the **tangent** (not outwards along the radius).

Equations

$$a = r\omega^2 = v^2 / r$$

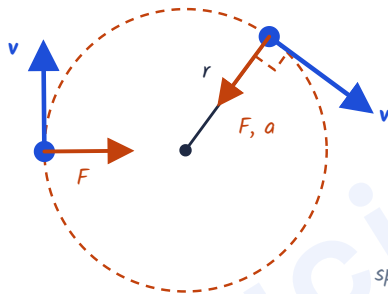
centripetal acceleration, towards the centre
 $m\ s^{-2}$

$$a = v\omega$$

using $v = r\omega$
 $m\ s^{-2}$

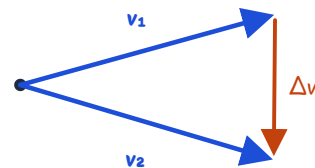
$$F = mr\omega^2 = mv^2 / r$$

centripetal (resultant) force, towards the centre
 N



Uniform circular motion: velocity is tangential; the centripetal force and acceleration point inwards along the radius.

$|v_1| = |v_2|$, directions differ



Δv (and so a) points to the centre

Why the acceleration points inwards: the change in velocity $\Delta v = v_2 - v_1$ between two nearby points is directed towards the centre.

Watch out!

- ! There is no outward "centrifugal force" on the object. Draw only the **real** forces; their resultant is inwards.
- ! In $F = mv^2/r$, F is the **resultant** force, not one of the forces added to it.

12.26 Applying circular motion

Key points

- ✓ Method: draw the real forces, find the **resultant towards the centre**, set it equal to mv^2/r .
- ✓ Car on a flat bend: **friction** between tyres and road provides the centripetal force.
- ✓ Banked track: the horizontal component of the **normal force** provides it; $N \cos \theta = mg$.
- ✓ Mass on a string in a vertical circle: tension is **greatest at the bottom**, least at the top.
- ✓ At the top, the string stays taut only if $v \geq \sqrt{gr}$; at this minimum speed $T = 0$ and gravity alone provides the force.
- ✓ Satellites and planets: **gravitational force** provides the centripetal force (see topic 13).
- ✓ Conical pendulum / aircraft banking: horizontal component of tension or lift provides it.

Equations

$$T_{\text{top}} + mg = mv^2 / r$$

vertical circle, top: both forces point to the centre

N

$$T_{\text{bottom}} - mg = mv^2 / r$$

vertical circle, bottom: tension up, weight down

N

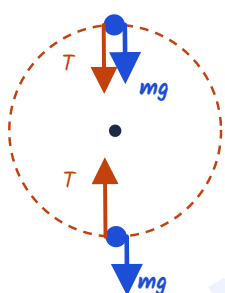
$$v_{\text{min}} = \sqrt{gr}$$

minimum speed at the top of a vertical circle

$m \text{ s}^{-1}$

$$\tan \theta = v^2 / rg$$

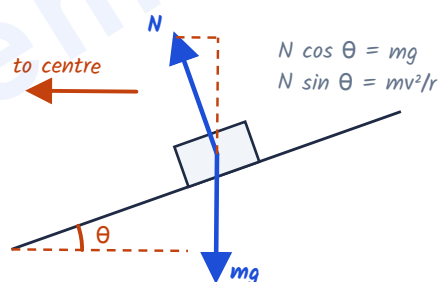
banked track with no friction needed



top:
 $T + mg = mv^2/r$

bottom:
 $T - mg = mv^2/r$

Vertical circle: at the top, tension and weight both act towards the centre; at the bottom, tension must exceed weight.



$N \cos \theta = mg$
 $N \sin \theta = mv^2/r$

Banked track: the horizontal component of the normal force, $N \sin \theta$, provides the centripetal force; $N \cos \theta$ balances the weight.

Watch out!

- ! At the top of a vertical circle, weight **helps** ($T + mg$); at the bottom it **opposes** ($T - mg$). Do not use the same equation for both.
- ! Use the radius to the **centre of mass** of the object, e.g. string length plus the ball radius if needed.