

Deformation of solids

A-Level Physics 9702 · Topic 6 revision notes · AS

Springs and wires under load: Hooke's law, stress, strain and the Young modulus, elastic and plastic behaviour, and the energy stored.

11 definitions to learn word for word, 7 equations in their own column, 5 diagrams and graphs. Read it, cover it, test yourself.

1. 6.1a Hooke's law and the spring constant
2. 6.1b Stress, strain and the Young modulus
3. 6.2 Elastic and plastic behaviour

Definitions (mark-scheme wording) Equations Exam traps

6.1a Hooke's law and the spring constant

Definitions

Hooke's law: the extension of a spring (or wire) is directly proportional to the applied force, provided the limit of proportionality is not exceeded.

Spring constant: the force per unit extension ($k = F/x$).

Limit of proportionality: the point beyond which force (stress) is no longer proportional to extension (strain).

Tensile / compressive force: a pair of forces that stretches / squashes an object.

Equations

$$F = kx$$

Hooke's law (up to the limit of proportionality)

N

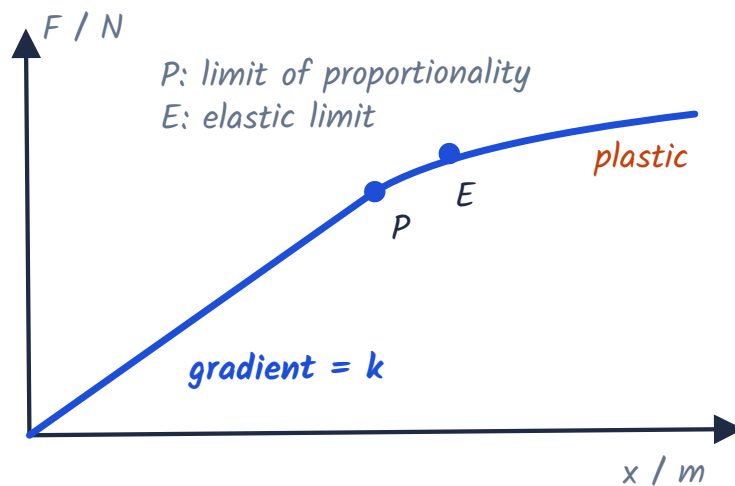
$$k = F / x$$

spring constant

N m⁻¹

Key points

- ✓ Force-extension graph: straight line through the origin up to the **limit of proportionality**.
- ✓ **Gradient** of F against $x = k$; a stiffer spring has a steeper line.
- ✓ Extension $x =$ stretched length - **original** length (not the total length).
- ✓ Compression works the same way: x is the decrease in length.
- ✓ k depends on the sample (material, length, thickness), not only on the material.



Watch out!

- ! Using the total length instead of the **extension** in $F = kx$.
- ! Assuming the limit of proportionality and the elastic limit are the same point; E is usually just beyond P.

6.1b Stress, strain and the Young modulus

Definitions

Tensile stress: the force per unit cross-sectional area (acting normal to the area).

Tensile strain: the extension per unit original length.

Young modulus: the ratio of tensile stress to tensile strain (within the limit of proportionality).

Equations

$$\sigma = F / A$$

tensile stress = force / cross-sectional area
Pa

$$\epsilon = x / L$$

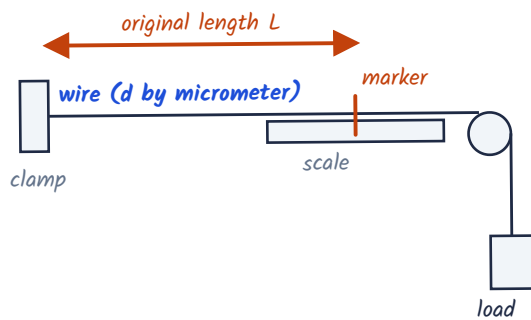
tensile strain = extension / original length
no unit

$$E = \sigma / \epsilon = FL / Ax$$

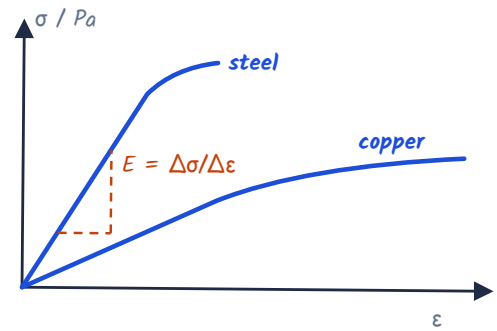
Young modulus
Pa

Key points

- ✓ Young modulus is a property of the **material**, independent of sample size and shape.
- ✓ Strain has **no unit**; stress and Young modulus are in **Pa**.
- ✓ Stress-strain graph: **gradient of the linear region = E**.
- ✓ Wire area: $A = \pi d^2 / 4$; measure d with a **micrometer** at several places and orientations, take the mean.
- ✓ Experiment: long, thin wire (large, measurable extension); measure L with a metre rule, x with a vernier/travelling microscope.
- ✓ Plot F against x : **$E = \text{gradient} \times L / A$** .



Measuring E : a long wire loaded over a pulley; a marker against a fixed scale gives the extension.



Stress-strain graphs: gradient in the linear region = Young modulus. Steel is stiffer than copper.

Watch out!

- ! Using the diameter instead of the radius (or forgetting to square) in $A = \pi r^2$.
- ! Forgetting unit conversions: mm to m, mm^2 to m^2 ($\times 10^{-6}$). E for metals is $\sim 10^{11}$ Pa.

6.2 Elastic and plastic behaviour

Definitions

Elastic deformation: deformation in which the object returns to its original shape and size when the load is removed.

Plastic deformation: deformation in which the object does not return to its original shape when the load is removed (permanent deformation).

Elastic limit: the maximum force (stress) beyond which the object no longer returns to its original length when the load is removed.

Elastic potential (strain) energy: the energy stored in a deformed object because of its change in shape.

Equations

$$E_p = \frac{1}{2}Fx$$

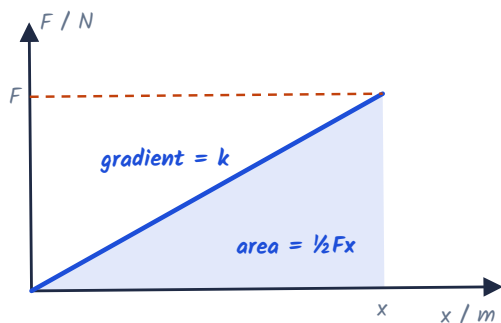
strain energy = area under a linear F - x graph
J

$$E_p = \frac{1}{2}kx^2$$

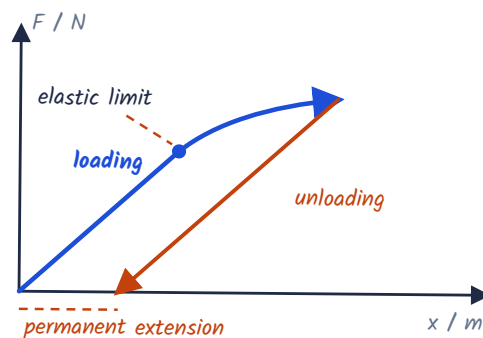
strain energy of a spring obeying Hooke's law
J

Key points

- ✓ Beyond the elastic limit, unloading leaves a **permanent extension**; the unloading line is parallel to the loading line.
- ✓ **Strain energy = area under the F - x graph** (work done to deform it).
- ✓ Linear region only: area is a triangle, $E_p = \frac{1}{2}Fx = \frac{1}{2}kx^2$.
- ✓ Non-linear graph: find the area by **counting squares** or splitting into shapes.
- ✓ Plastic deformation: area between loading and unloading curves = energy **dissipated as thermal energy**.
- ✓ Doubling the extension (within the limit) **quadruples** the stored energy.



Strain energy stored = shaded triangle under the F - x line = $\frac{1}{2}Fx$.



Loaded past the elastic limit; unloading runs parallel to the loading line and leaves a permanent extension.

Watch out!

- ! $\frac{1}{2}Fx$ and $\frac{1}{2}kx^2$ apply only to the **linear** region; beyond it use the area under the curve.
- ! Using the total length instead of the extension, or forgetting the $\frac{1}{2}$.