

Gravitational fields

A-Level Physics 9702 · Topic 13 revision notes · A2

Newton's inverse-square law of gravitation, field strength and potential of a point mass, and the physics of circular orbits and geostationary satellites.

11 definitions to learn word for word, 13 equations in their own column, 10 diagrams and graphs. Read it, cover it, test yourself.

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2. 13.2a Newton's law of gravitation
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Definitions (mark-scheme wording) Equations Exam traps

13.1 Gravitational field

Definitions

Gravitational field: a region of space in which a mass experiences a force.

Gravitational field strength (g): the gravitational force per unit mass acting on a small mass placed at that point.

Field line: a line showing the direction of the force on a small mass placed in the field.

Equations

$$g = F / m$$

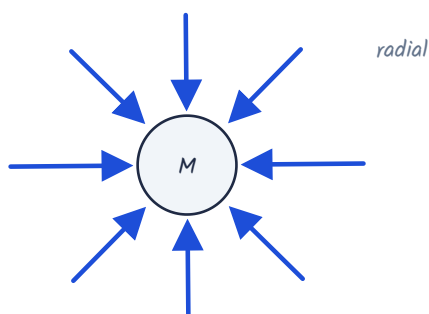
field strength = force per unit mass
 N kg^{-1}

$$W = mg$$

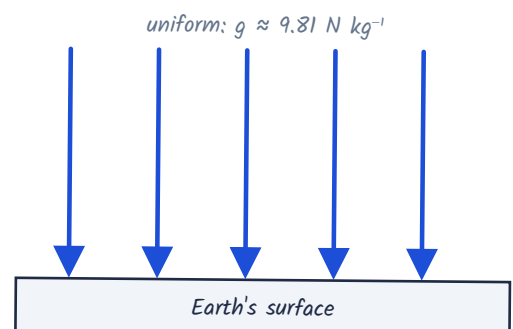
weight of a mass m in field g
 N

Key points

- ✓ g is a **vector**, directed towards the mass producing the field.
- ✓ Field lines always point **towards** masses: gravity is only attractive.
- ✓ Closer field lines mean a **stronger** field.
- ✓ Around a point or spherical mass the field is **radial**; near the Earth's surface it is effectively **uniform** (parallel, equally spaced lines).
- ✓ N kg^{-1} and m s^{-2} are equivalent units for g .



Radial field around a spherical mass: lines point to the centre and crowd together near the surface, where the field is strongest.



Near the surface the field is uniform: parallel, equally spaced lines, so g is the same everywhere in the region.

Watch out!

- ! Gravitational field lines never point outwards; there is no gravitational repulsion.
- ! g is force **per unit mass**, not the force itself.

13.2a Newton's law of gravitation

Definitions

Newton's law of gravitation: any two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation.

Point mass: a mass whose size is negligible compared with the distances involved; a uniform sphere acts as a point mass at its centre.

Equations

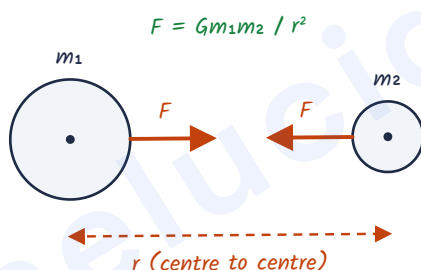
$$F = Gm_1m_2 / r^2$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

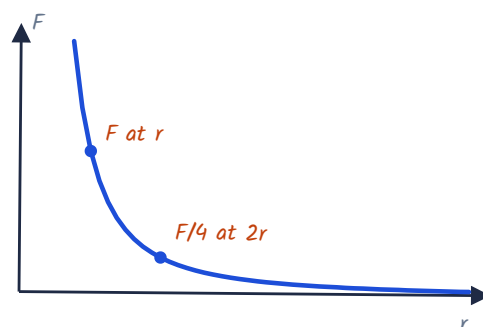
N

Key points

- ✓ For uniform spheres, measure r **centre to centre**.
- ✓ The forces on the two masses are **equal and opposite** (Newton's third law), even if the masses differ.
- ✓ Inverse square: double r , force falls to **one quarter**.
- ✓ G is tiny, so gravity is only noticeable when at least one mass is very large.



Two masses attract with equal and opposite forces; r is measured between the centres.



Inverse-square law: F against r falls steeply; each doubling of r quarters the force.

Watch out!

- ! r is the distance between **centres**, not the height above the surface.
- ! Forgetting to square r : the force depends on $1/r^2$.

13.26 Circular orbits and geostationary satellites

Definitions

Geostationary orbit: an equatorial orbit, travelling west to east, with a period of 24 hours, so the satellite stays above the same point on the Earth's surface.

Orbital period (T): the time taken for one complete orbit.

Key points

- ✓ For a circular orbit, **gravitational force provides the centripetal force**: $GMm/r^2 = mv^2/r$.
- ✓ The satellite mass **m cancels**: orbital speed and period depend only on M and r.
- ✓ Larger orbit: **slower speed, longer period**.
- ✓ $T^2 \propto r^3$ (Kepler's third law): plot T^2 against r^3 , gradient = $4\pi^2/GM$, so M can be found.
- ✓ Geostationary radius $\approx 4.2 \times 10^7$ m (about 3.6×10^7 m above the surface).
- ✓ Geostationary satellites suit communications and weather monitoring: dishes stay fixed; but coverage near the poles is poor.

Equations

$$GMm/r^2 = mv^2/r$$

gravitational force = centripetal force
N

$$v = \sqrt{GM/r}$$

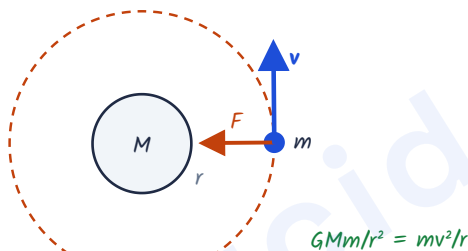
orbital speed
 $m\ s^{-1}$

$$v = 2\pi r / T$$

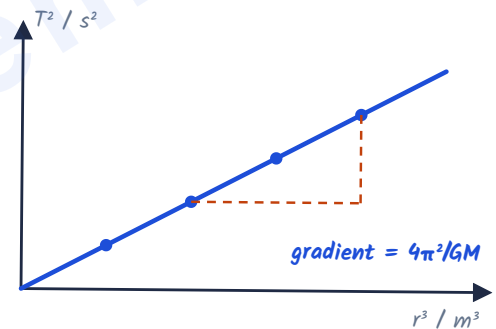
speed from orbit circumference and period
 $m\ s^{-1}$

$$T^2 = (4\pi^2 / GM) r^3$$

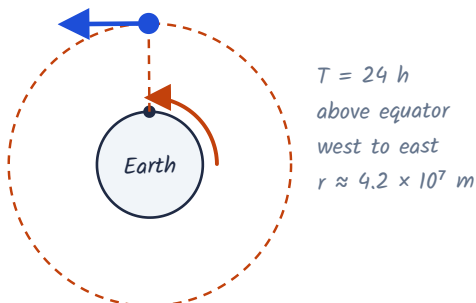
Kepler's third law for circular orbits
 s^2



Circular orbit: the gravitational pull is the centripetal force, always towards the planet's centre, perpendicular to v.



Kepler's third law: T^2 against r^3 is a straight line through the origin with gradient $4\pi^2/GM$.



Geostationary satellite (viewed from above the North Pole): it orbits in the same direction as the Earth spins, with the same 24 h period.

Watch out!

- ! Use $r = R + h$ (orbit radius from the centre) in orbit equations, not the height h .
- ! Geostationary needs **all three**: equatorial, west to east, 24 h period. A 24 h polar orbit is not geostationary.

13.3 Gravitational field of a point mass

Definitions

Gravitational field strength of a point mass: $g = GM/r^2$, directed towards the mass, where r is the distance from its centre.

Equations

$$g = GM / r^2$$

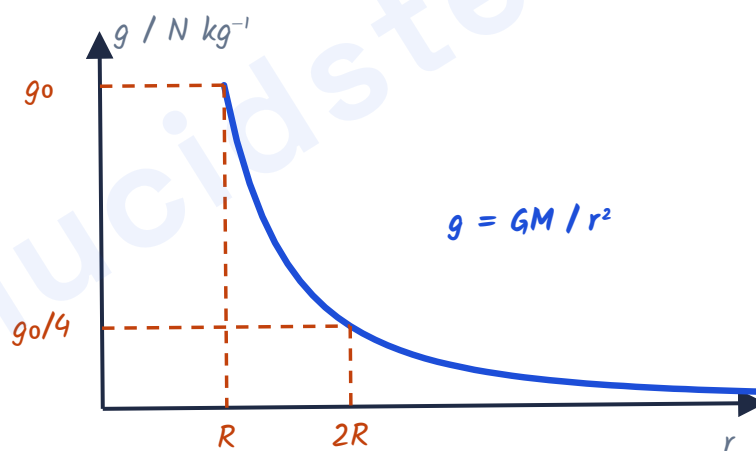
field strength of a point (or spherical) mass
 N kg^{-1}

$$g_0 = GM / R^2$$

field at the surface of a planet of radius R
 N kg^{-1}

Key points

- ✓ Derived from $g = F/m$ with $F = GMm/r^2$: the test mass m cancels.
- ✓ Outside a uniform sphere, g follows the **inverse-square law**: double r , g falls to a quarter.
- ✓ Near the Earth's surface r hardly changes over everyday heights, so g is **approximately constant** (9.81 N kg^{-1}).
- ✓ g at altitude compared with the surface: $g/g_0 = R^2/r^2$.
- ✓ Useful link: $GM = g_0 R^2$ (surface field and planet radius).



Field strength outside a planet: g_0 at the surface ($r = R$), falling to $g_0/4$ at $r = 2R$.

Watch out!

- ! g falls as $1/r^2$, not $1/r$: tripling the distance from the centre cuts g to a ninth.
- ! $g = GM/r^2$ applies only **outside** (or at the surface of) the sphere.

13.4 Gravitational potential and potential energy

Definitions

Gravitational potential (ϕ): the work done per unit mass in bringing a small test mass from infinity to that point.

Gravitational potential energy (of a mass m at a point): the work done in bringing the mass from infinity to that point.

Equipotential: a surface on which every point has the same potential; no work is done moving a mass along it.

Key points

- ✓ Zero of potential is at **infinity**, and ϕ is **negative everywhere** because gravity is attractive (work is done by the field as mass moves in).
- ✓ Moving away from a mass, ϕ **increases** (becomes less negative).
- ✓ Change in potential energy: $\Delta E_p = m\Delta\phi = GMm(1/r_1 - 1/r_2)$ moving from r_1 out to r_2 .
- ✓ Near the surface only, $\Delta E_p = mg\Delta h$ is a good approximation (g constant).
- ✓ Field strength = **minus the potential gradient**: $g = -\Delta\phi/\Delta r$ (gradient of the ϕ - r graph).
- ✓ Equipotentials around a point mass are **concentric spheres**, perpendicular to the field lines.

Equations

$$\phi = -GM/r$$

potential of a point mass, zero at infinity
 J kg^{-1}

$$E_p = -GMm/r$$

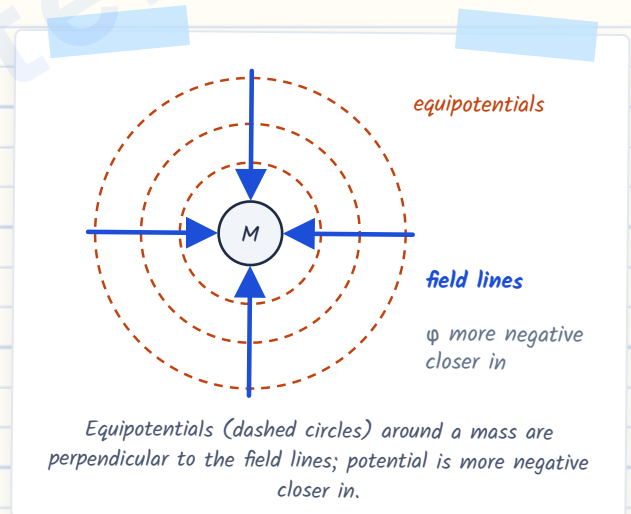
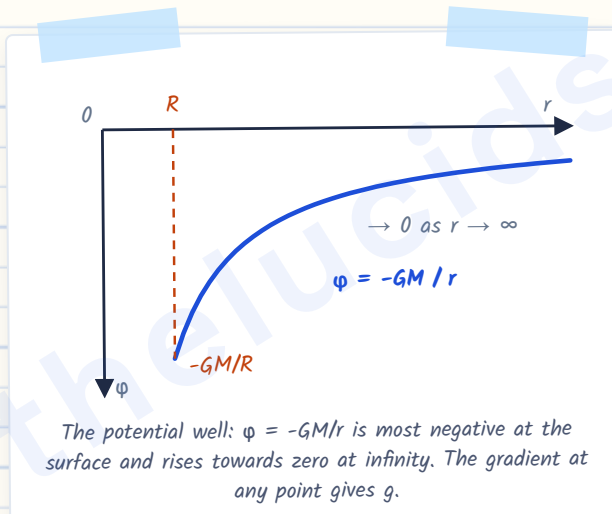
potential energy of mass m at distance r
 J

$$\Delta E_p = m\Delta\phi$$

change in potential energy
 J

$$g = -\Delta\phi/\Delta r$$

field strength = minus potential gradient
 N kg^{-1}



Watch out!

- ! A more negative ϕ is a **lower** potential: $-6 \times 10^7 \text{ J kg}^{-1}$ is lower than $-3 \times 10^7 \text{ J kg}^{-1}$.
- ! Use $mg\Delta h$ only for small height changes near the surface; use $m\Delta\phi$ for satellites.