

Ideal gases

A-Level Physics 9702 · Topic 15 revision notes · A2

The mole, the ideal gas equation of state, and the kinetic theory that links gas pressure and temperature to molecular motion.

12 definitions to learn word for word, 13 equations in their own column, 7 diagrams and graphs. Read it, cover it, test yourself.

1. 15.1 The mole
2. 15.2 Equation of state
3. 15.3a Kinetic theory: assumptions and pressure
4. 15.3b Kinetic energy of a molecule and temperature

Definitions (mark-scheme wording) Equations Exam traps

15.1 The mole

Definitions

Amount of substance: an SI base quantity, measured in moles (mol).

Mole: the amount of substance containing 6.02×10^{23} particles (the Avogadro constant number of particles).

Avogadro constant (N_A): the number of particles in one mole of a substance: $6.02 \times 10^{23} \text{ mol}^{-1}$.

Molar mass (M): the mass of one mole of a substance.

Equations

$$N = nN_A$$

number of particles = moles $\times$ Avogadro constant
(no unit)

$$n = \text{mass} / M$$

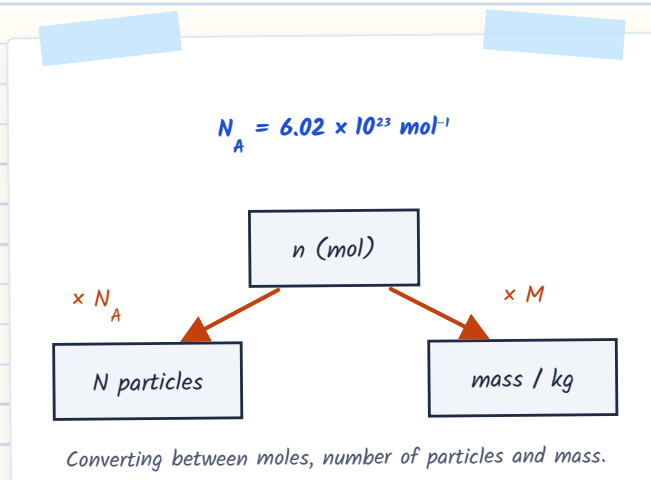
moles from mass and molar mass
mol

$$m = M / N_A$$

mass of one molecule
kg

Key points

- ✓ Number of molecules $N = nN_A$; mass of gas = nM .
- ✓ Molar mass in g mol^{-1} equals the nucleon number: **convert to kg mol^{-1}** before use.
- ✓ Mass of one molecule $m = M / N_A$.
- ✓ One mole of **any** substance has the same number of particles, whatever its mass.



Watch out!

- ! n (moles) and N (molecules) differ by a factor of N_A : $pV = nRT$ uses n , $pV = NkT$ uses N .
- ! Molar mass of helium is $4 \text{ g mol}^{-1} = 0.004 \text{ kg mol}^{-1}$.

15.2 Equation of state

Definitions

Ideal gas: a gas that obeys $pV \propto T$ (or $pV = nRT$) at all pressures, volumes and temperatures.

Boyle's law: for a fixed mass of gas at constant temperature, pressure is inversely proportional to volume.

Charles's law: for a fixed mass of gas at constant pressure, volume is proportional to thermodynamic temperature.

Pressure law: for a fixed mass of gas at constant volume, pressure is proportional to thermodynamic temperature.

Equations

$$pV = nRT$$

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\text{Pa m}^3 = \text{J}$$

$$pV = NkT$$

$$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\text{Pa m}^3 = \text{J}$$

$$k = R / N_A$$

Boltzmann constant

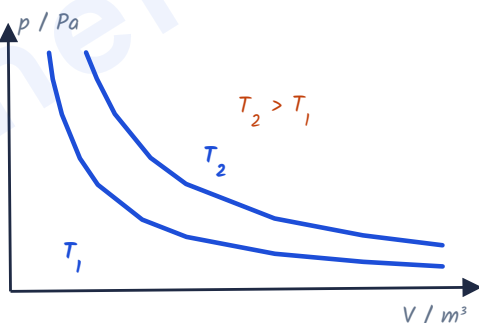
$$\text{J K}^{-1}$$

$$p_1 V_1 / T_1 = p_2 V_2 / T_2$$

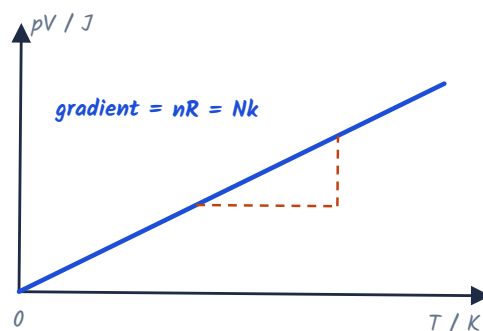
fixed mass of gas, before and after
(any consistent units, T in K)

Key points

- ✓ The three gas laws combine to $pV / T = \text{constant}$ for a fixed mass of ideal gas.
- ✓ Molar form uses the **molar gas constant R** ; molecular form uses the **Boltzmann constant $k = R / N_A$** .
- ✓ Real gases behave most ideally at **low pressure** and **high temperature** (well above boiling point).
- ✓ Isotherms on a p - V graph are hyperbolas; a higher temperature gives a curve further from the axes.
- ✓ A graph of pV against T is a straight line through the origin with **gradient nR** .



Boyle's law isotherms: $p \propto 1/V$ at constant T ; the hotter isotherm lies further out.



pV against T (kelvin): straight line through the origin, gradient $nR = Nk$.

Watch out!

- ! Use p in Pa, V in m^3 ($1 \text{ cm}^3 = 10^{-6} \text{ m}^3$, $1 \text{ litre} = 10^{-3} \text{ m}^3$) and T in K.
- ! The gas laws need a **fixed mass** of gas: if gas leaks, n changes.

15.3a Kinetic theory: assumptions and pressure

Definitions

Kinetic theory assumptions: a large number of identical molecules in random motion; negligible volume of molecules compared with the container; negligible intermolecular forces except during collisions; perfectly elastic collisions; collision time negligible compared with time between collisions.

Pressure (of a gas): the force per unit area exerted on the walls due to the rate of change of momentum of molecules colliding with them.

Equations

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

kinetic theory equation; $\langle c^2 \rangle$ = mean-square speed

$$\text{Pa m}^3 = \text{J}$$

$$p = \frac{1}{3}\rho\langle c^2 \rangle$$

same result using density

$$\text{Pa}$$

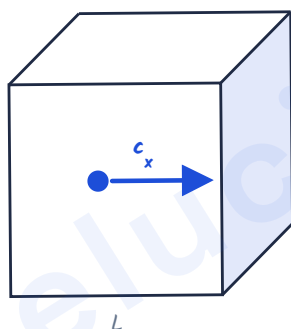
$$F = \Delta(mv) / \Delta t$$

force from one molecule = mc_x^2 / L

$$\text{N}$$

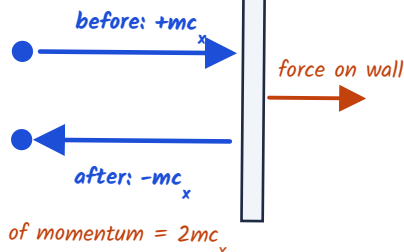
Key points

- ✓ Molecule of mass m hits a wall with velocity component c_x :
momentum change = $2mc_x$.
- ✓ In a cube of side L it returns to the same wall after time $2L / c_x$.
- ✓ Force = rate of change of momentum = $2mc_x / (2L / c_x) = mc_x^2 / L$.
- ✓ For N molecules, $p = F / L^2 = Nm\langle c_x^2 \rangle / V$.
- ✓ Random motion: $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$, so $\langle c_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$.
- ✓ Since density $\rho = Nm / V$, the result can also be written $p = \frac{1}{3}\rho\langle c^2 \rangle$.



at wall:
 $\Delta p = 2mc_x$
 returns in
 $t = 2L / c_x$
 $F = mc_x^2 / L$

Derivation set-up: one molecule in a cube of side L bouncing between opposite faces.



At the wall the molecule's momentum reverses: change $2mc_x$ and the wall feels an equal and opposite push.

Watch out!

- ! The momentum change at the wall is $2mc_x$, not mc_x ; the velocity reverses.
- ! $\langle c^2 \rangle$ is the **mean of the squared speeds**, not the square of the mean speed.

15.36 Kinetic energy of a molecule and temperature

Definitions

Root-mean-square speed ($c_{r.m.s.}$): the square root of the mean of the squares of the speeds of the molecules.

Boltzmann constant (k): the molar gas constant divided by the Avogadro constant, $k = R / N_A$.

Key points

- ✓ Equate $pV = \frac{1}{2}Nm\langle c^2 \rangle$ with $pV = NkT$ to get $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$.
- ✓ Mean translational kinetic energy of a molecule is **proportional to thermodynamic temperature**.
- ✓ At the same temperature all ideal gases have the **same mean kinetic energy** per molecule; lighter molecules move faster.
- ✓ $c_{r.m.s.} = \sqrt{3kT / m}$: doubling T multiplies $c_{r.m.s.}$ by $\sqrt{2}$.
- ✓ r.m.s. speed: square each speed, take the mean, then the square root.
- ✓ At higher T the speed distribution spreads out and its peak shifts to higher speed.

Equations

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

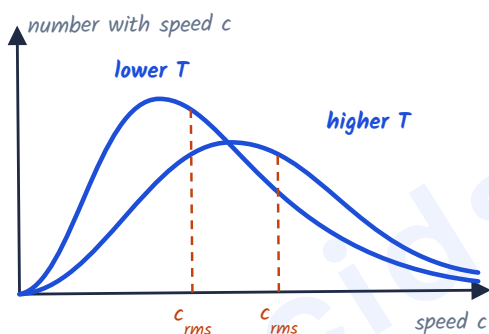
mean translational kinetic energy of one molecule
J

$$c_{r.m.s.} = \sqrt{\langle c^2 \rangle}$$

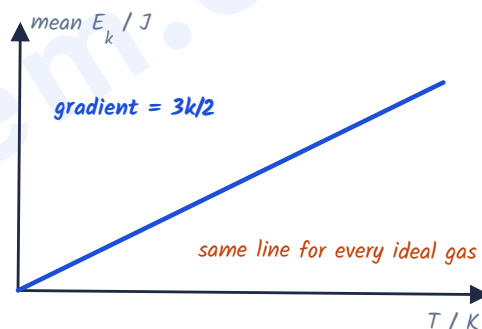
root-mean-square speed
 $m\ s^{-1}$

$$c_{r.m.s.} = \sqrt{3kT / m}$$

from the kinetic energy equation
 $m\ s^{-1}$



Molecular speed distributions: at higher T the peak moves right and flattens; $c_{r.m.s.} \propto \sqrt{T}$.



Mean kinetic energy per molecule against T : straight line through the origin, gradient $3k/2$.

Watch out!

- ! $c_{r.m.s.}$ is **larger** than the mean speed; do not average speeds and then square.
- ! Use the mass of **one molecule** (M / N_A , in kg) in $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$.