

Medical physics

A-Level Physics 9702 · Topic 24 revision notes · A2

Imaging inside the body: ultrasound pulse-echo and acoustic impedance, X-ray production, attenuation and CT, and PET scanning with positron annihilation.

14 definitions to learn word for word, 12 equations in their own column, 10 diagrams and graphs. Read it, cover it, test yourself.

1. 24.1a Ultrasound: production and pulse-echo imaging.
2. 24.1b Acoustic impedance, reflection and attenuation
3. 24.2a Production of X-rays
4. 24.2b X-ray imaging, contrast and CT scanning.
5. 24.3 PET scanning.

■ Definitions (mark-scheme wording) ■ Equations ■ Exam traps

24.1a Ultrasound: production and pulse-echo imaging

Definitions

Ultrasound: longitudinal (sound) waves with frequency above the upper limit of human hearing, about 20 kHz.

Piezoelectric effect: a p.d. applied across certain crystals (e.g. quartz) makes them change shape; conversely, deforming the crystal produces a p.d.

Transducer: a device that converts energy from one form to another; here it both emits and receives ultrasound.

Equations

$$d = \frac{1}{2}ct$$

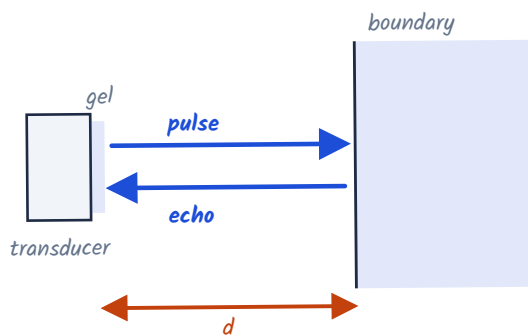
depth of boundary from echo time t (there and back)
 m

$$c = f\lambda$$

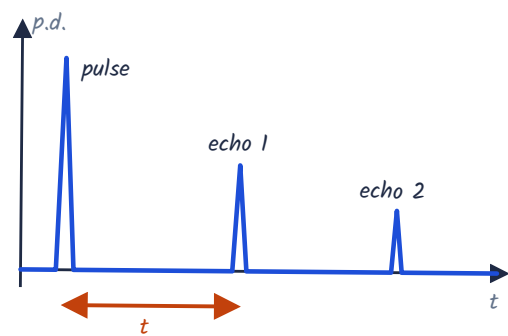
speed of the ultrasound in tissue
 $m\ s^{-1}$

Key points

- ✓ **Production:** an alternating p.d. makes the piezoelectric crystal vibrate; at its **resonant frequency** the amplitude is largest.
- ✓ **Detection:** a returning echo makes the crystal vibrate, generating a small alternating p.d.
- ✓ The same transducer sends a **short pulse**, then listens for echoes before the next pulse.
- ✓ Echoes come from **boundaries** between tissues; the time delay gives the depth of each boundary.
- ✓ Medical ultrasound uses about **1 to 10 MHz**: short wavelength gives fine detail.
- ✓ Safe (non-ionising); good for soft tissue and a fetus, poor through bone or air.



Pulse-echo: the transducer emits a pulse, part reflects at each boundary, and the echo time gives the depth $d = \frac{1}{2}ct$.



Echo display (A-scan): time between the transmitted pulse and each echo gives the depth of each boundary.

Watch out!

- ! The echo goes **there and back**: $d = \frac{1}{2}ct$, not ct .
- ! The crystal emits at its **resonant** frequency; the pulse must end before the echo returns.

24.16 Acoustic impedance, reflection and attenuation

Definitions

Specific acoustic impedance (Z): the product of the density of a medium and the speed of sound in it: $Z = \rho c$.

Intensity reflection coefficient: the ratio of the reflected intensity to the incident intensity at a boundary.

Attenuation: the gradual decrease in intensity of a beam as it passes through a medium (by absorption and scattering).

Linear attenuation coefficient (μ): the constant in $I = I_0 e^{-\mu x}$; the fractional decrease in intensity per unit thickness.

Equations

$$Z = \rho c$$

specific acoustic impedance
 $\text{kg m}^{-2} \text{s}^{-1}$

$$I_R/I_0 = (Z_2 - Z_1)^2 / (Z_2 + Z_1)^2$$

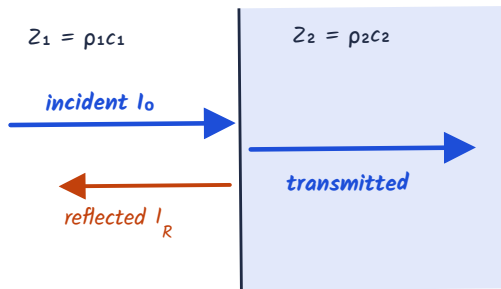
intensity reflection coefficient
no unit

$$I = I_0 e^{-\mu x}$$

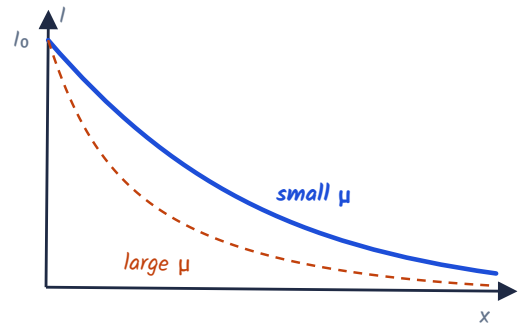
attenuation through thickness x
 W m^{-2}

Key points

- ✓ Reflection at a boundary depends on the **difference in Z** : big difference, strong reflection.
- ✓ Soft tissue to soft tissue: small difference, weak echo, most passes on to deeper boundaries.
- ✓ Air-skin: huge difference, nearly **total reflection**, so **coupling gel** (Z close to skin) removes the air gap.
- ✓ Intensity falls **exponentially** with depth; higher frequency attenuates more, so trade detail against depth.
- ✓ Echoes from deep boundaries are weaker: they suffer attenuation both ways and partial reflection at each boundary.



At a boundary the incident intensity splits into reflected and transmitted parts; I_R depends on $(Z_2 - Z_1)^2$.



Attenuation: intensity falls exponentially with thickness x ; larger μ gives a steeper fall.

Watch out!

- ! Use Z values, not densities alone, in the reflection coefficient; the difference is **squared**.
- ! Gel is needed because air-skin reflects almost everything, not because gel "conducts" sound better.

24.2a Production of X-rays

Definitions

X-rays: high-energy (short-wavelength) electromagnetic radiation produced when high-speed electrons are decelerated by a metal target.

Braking radiation (bremsstrahlung): the continuous spectrum of X-rays emitted as electrons decelerate in the target.

Equations

$$eV = hc / \lambda_{\min}$$

minimum wavelength from accelerating p.d. V

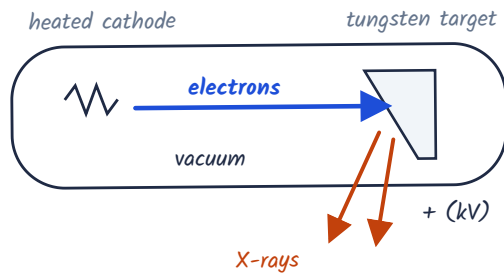
J

$$\lambda_{\min} = hc / (eV)$$

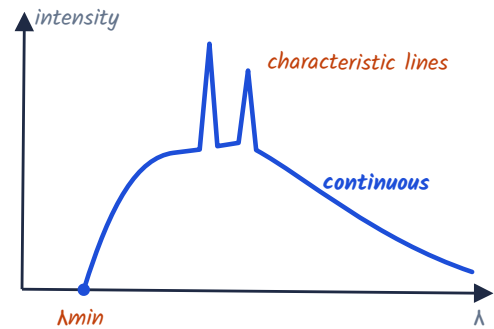
shortest X-ray wavelength
 m

Key points

- ✓ Electrons from a **heated cathode** (thermionic emission) are accelerated through a large p.d. (tens of kV) to a metal **anode target** (e.g. tungsten).
- ✓ Only about **1%** of the electron energy becomes X-rays; the rest heats the target, so it is cooled (rotating anode, oil, high melting point metal).
- ✓ The spectrum is **continuous** up to a maximum photon energy, with **characteristic lines** from inner-shell electron transitions in the target.
- ✓ Maximum photon energy = eV , when an electron loses all its KE in one interaction: this sets $\lambda_{\min}$.
- ✓ Higher tube p.d.: smaller $\lambda_{\min}$, more penetrating (**harder**) X-rays. Higher tube current: more X-rays (greater intensity).
- ✓ An aluminium filter removes **low-energy** X-rays that would only be absorbed by the skin.



X-ray tube: electrons from the heated cathode hit the tungsten target and X-rays leave through a window.



X-ray spectrum: continuous braking radiation from $\lambda_{min} = hc/eV$, with sharp characteristic lines of the target metal.

Watch out!

- ! Tube p.d. sets the **shortest** wavelength (maximum energy); tube current sets intensity, not λ_{min} .
- ! Characteristic line positions depend on the **target metal**, not on the p.d.

24.26 X-ray imaging, contrast and CT scanning

Definitions

Contrast: the difference in degree of blackening between neighbouring regions of an image (between structures).

CT scan (computed tomography): X-ray images of many thin slices, taken from many different directions, combined by computer to build a three-dimensional image.

Equations

$$I = I_0 e^{-\mu x}$$

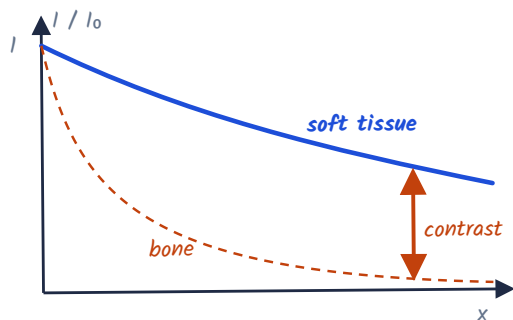
X-ray attenuation; μ depends on tissue and photon energy
 W m^{-2}

$$\ln(I/I_0) = -\mu x$$

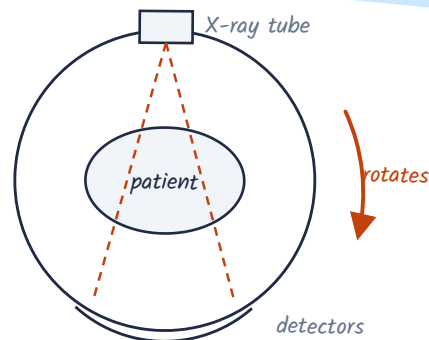
log form for finding μ or x
 no unit

Key points

- ✓ Image = X-ray **shadow**: bone attenuates strongly (large μ), soft tissue weakly.
- ✓ Good contrast needs **very different** μ values; soft tissues give poor contrast.
- ✓ Contrast media (barium meal, iodine) have large μ to show soft organs such as the gut or blood vessels.
- ✓ Sharpness improves with a small source (focal spot) and a collimated beam.
- ✓ **CT**: tube and detectors rotate round the patient; each slice is built from many 1-D projections, then slices stack into a 3-D image.
- ✓ CT gives 3-D detail and better soft-tissue contrast, but a much **larger radiation dose**.
- ✓ X-rays are **ionising**: keep doses as low as possible.



Bone has a much larger μ than soft tissue, so far less X-ray gets through: the difference gives contrast.



CT: the X-ray tube and opposite detector array rotate around the patient, recording one slice from many angles.

Watch out!

- ! Contrast depends on the **difference** in μ , not on μ itself.
- ! CT is not one X-ray photo: it reconstructs **slices** from many angles, giving 3-D information.

24.3 PET scanning

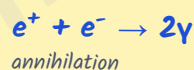
Definitions

Tracer: a substance containing a radioactive nuclide that is introduced into the body and absorbed by the tissue being studied.

Annihilation: the process in which a particle and its antiparticle (e.g. an electron and a positron) meet and their total mass-energy is converted into photons.

PET scanning: positron emission tomography: imaging the distribution of a β^+ -emitting tracer by detecting pairs of gamma photons from annihilation.

Equations



$$E = m_e c^2$$

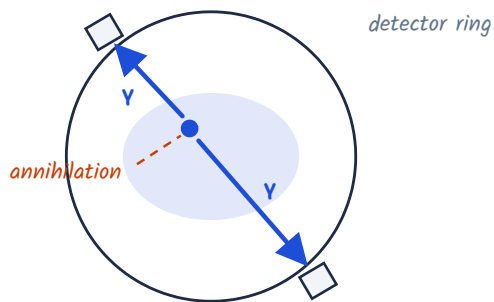
energy of each gamma photon (0.511 MeV
= 8.2×10^{-14} J)

$$2m_e c^2 = 2hf$$

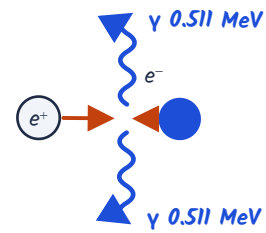
total energy of the photon pair

Key points

- ✓ Tracer (e.g. Fluorine-18 in a glucose analogue) collects where **metabolic activity** is high (e.g. tumours, active brain regions).
- ✓ The nuclide decays by **β^+ emission**; the positron travels ~ 1 mm, then **annihilates** with an electron.
- ✓ Two **gamma photons** leave in **opposite directions** (momentum conserved, initial momentum ~ 0).
- ✓ Each photon has energy = rest energy of an electron = **0.511 MeV** (kinetic energies negligible).
- ✓ A ring of detectors records photons arriving **at the same time**; the annihilation lies on the line joining the two detectors.
- ✓ Arrival-time difference fixes the point on that line; many events build a **3-D image** of tracer concentration.
- ✓ PET shows **function** (metabolism), not just structure; often combined with CT.



PET: an annihilation sends two 0.511 MeV gamma photons back-to-back to detectors on the ring; the line between them passes through the source.



momentum before ≈ 0
Positron-electron annihilation: mass converts to two photons of equal energy moving in opposite directions.

Watch out!

- ! The two photons go in **opposite** directions (180° apart); that is what places the source on a line.
- ! Photon energy is 0.511 MeV **each** (not 1.02 MeV each): the pair shares $2m_e c^2$.
- ! The patient receives β^+ -emitting tracer; the **gamma** photons are what is detected, not the positrons.