

Oscillations

A-Level Physics 9702 · Topic 17 revision notes · A2

Simple harmonic motion defined by $a = -\omega^2 x$, its displacement, velocity and acceleration graphs, energy interchange, damping and resonance.

16 definitions to learn word for word, 15 equations in their own column, 12 diagrams and graphs. Read it, cover it, test yourself.

1. 17.1a Describing oscillations
2. 17.1b Simple harmonic motion: definition and equations
3. 17.1c Graphs of displacement, velocity and acceleration
4. 17.2 Energy in simple harmonic motion
5. 17.3a Damped oscillations
6. 17.3b Forced oscillations and resonance

■ Definitions (mark-scheme wording) ■ Equations ■ Exam traps

17.1a Describing oscillations

Definitions

Displacement (x): the distance from the equilibrium position in a specified direction.

Amplitude (x_0): the maximum displacement from the equilibrium position.

Period (T): the time taken for one complete oscillation.

Frequency (f): the number of oscillations per unit time.

Angular frequency (ω): the rate of change of phase angle; $\omega = 2\pi f$.

Phase difference: the fraction of a cycle between two oscillations, expressed as an angle (2π rad = one cycle).

Equations

$$f = 1 / T$$

frequency from period
Hz

$$\omega = 2\pi f = 2\pi / T$$

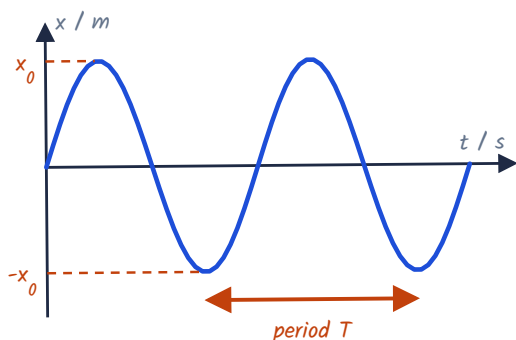
angular frequency
rad s⁻¹

$$\phi = 2\pi \Delta t / T$$

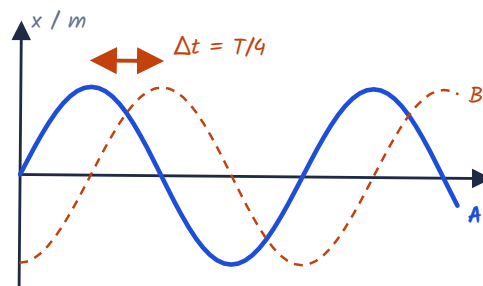
phase difference from time lag
rad

Key points

- ✓ One complete oscillation corresponds to a phase change of **2π rad** (360°).
- ✓ Two oscillations with the same frequency and a time lag Δt have **phase difference $2\pi \Delta t / T$** .
- ✓ In phase: difference 0 (or 2π); in **antiphase**: difference π rad.
- ✓ Angular frequency is measured in **rad s⁻¹**; it is not an angular speed of anything physically rotating.



Displacement-time graph: amplitude x_0 is the peak height; period T is one full cycle.



Oscillation B lags A by a quarter period: phase difference $\pi/2$ rad (90°).

Watch out!

- ! Phase difference in radians: use $2\pi\Delta t/T$, not $\Delta t/T$.
- ! Amplitude is measured from the **equilibrium position**, not peak to trough.

17.16 Simple harmonic motion: definition and equations

Definitions

Simple harmonic motion: motion in which the acceleration is proportional to the displacement from a fixed point and is always directed towards that point.

Key points

- ✓ Defining equation $a = -\omega^2 x$: the minus sign means acceleration is opposite to displacement.
- ✓ The **restoring force** is proportional to displacement and directed towards equilibrium.
- ✓ Period of SHM is **independent of amplitude** (isochronous).
- ✓ Starting at equilibrium: $x = x_0 \sin \omega t$ and $v = v_0 \cos \omega t$. Starting at an extreme: $x = x_0 \cos \omega t$.
- ✓ Maximum acceleration $\omega^2 x_0$ at the extremes; maximum speed ωx_0 at the centre.
- ✓ A graph of a against x is a straight line through the origin with **gradient $-\omega^2$** .

Equations

$$a = -\omega^2 x$$

defining equation of SHM
 $m s^{-2}$

$$x = x_0 \sin \omega t$$

displacement, timed from equilibrium
 m

$$v = v_0 \cos \omega t$$

velocity, timed from equilibrium
 $m s^{-1}$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

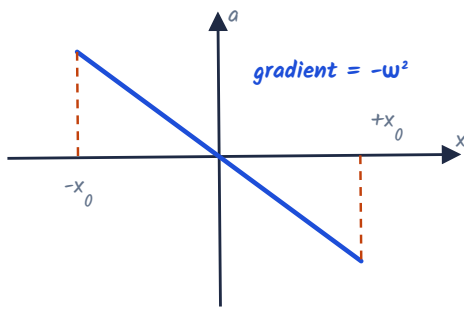
speed at displacement x
 $m s^{-1}$

$$v_0 = \omega x_0$$

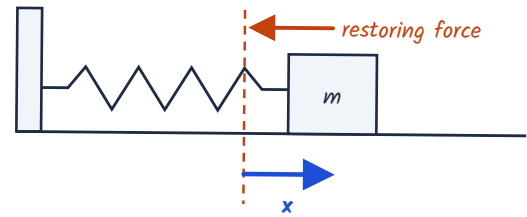
maximum speed (at $x = 0$)
 $m s^{-1}$

$$a_{\max} = \omega^2 x_0$$

maximum acceleration (at $x = \pm x_0$)
 $m s^{-2}$



a against x for SHM: straight line through the origin, gradient $-\omega^2$.



equilibrium position

Mass-spring oscillator: displaced by x , the restoring force points back towards equilibrium.

Watch out!

- ! Forgetting the **minus sign**: a and x are always in opposite directions.
- ! Use radians on the calculator for $\sin \omega t$ and $\cos \omega t$.

17.1c Graphs of displacement, velocity and acceleration

Definitions

Antiphase: a phase difference of π rad (half a cycle) between two oscillations.

Key points

- ✓ Velocity **leads** displacement by a quarter cycle ($\pi/2$ rad).
- ✓ Acceleration is in **antiphase** with displacement (π rad).
- ✓ v is the **gradient** of the x - t graph; a is the gradient of the v - t graph.
- ✓ At the extremes: $x = \pm x_0$, $v = 0$, $|a|$ maximum. At the centre: $x = 0$, $|v|$ maximum, $a = 0$.
- ✓ A v - x graph is an **ellipse**; its intercepts are $\pm x_0$ and $\pm v_0$.

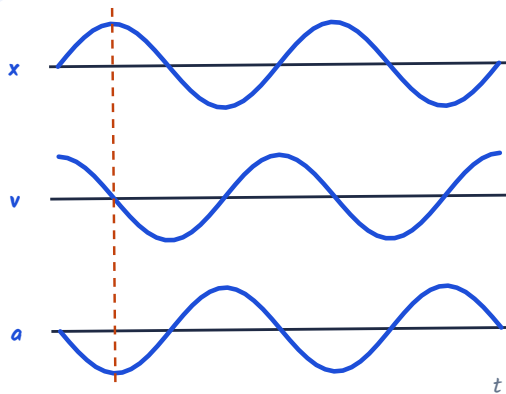
Equations

$$v = v_0 \cos \omega t$$

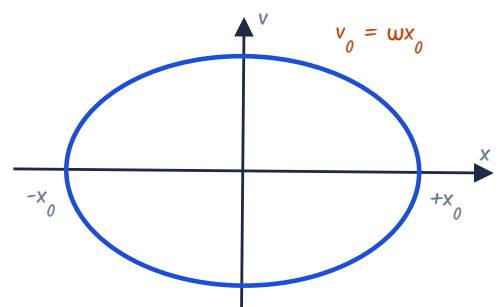
with $x = x_0 \sin \omega t$
 $m s^{-1}$

$$a = -\omega^2 x_0 \sin \omega t$$

acceleration against time
 $m s^{-2}$



x , v and a against time: the dashed line marks an extreme, where $v = 0$ and a is most negative.



v against x is an ellipse: $v = 0$ at $x = \pm x_0$, maximum speed $v_0 = \omega x_0$ at $x = 0$.

Watch out!

- ! Where x is maximum, v is **zero**, not maximum.
- ! Velocity and acceleration are not in phase: a is greatest where v is zero.

17.2 Energy in simple harmonic motion

Definitions

Energy interchange in SHM: kinetic energy and potential energy continuously interchange, the total energy remaining constant (no damping).

Key points

- ✓ At the centre: energy all **kinetic**. At the extremes: energy all **potential**.
- ✓ Total energy $\propto x_0^2$: doubling the amplitude quadruples the energy.
- ✓ $E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$; $E_p = \frac{1}{2}m\omega^2x^2$; they add to the constant total.
- ✓ Against time, E_k and E_p each vary at **twice** the oscillation frequency.
- ✓ Energy graphs against x are parabolas; against t they are $\sin^2$ and $\cos^2$ shapes, never negative.

Equations

$$E = \frac{1}{2}m\omega^2x_0^2$$

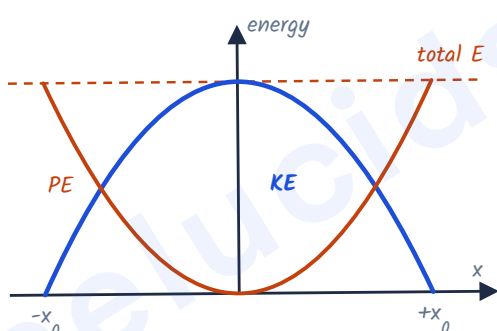
total energy of the oscillator
J

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

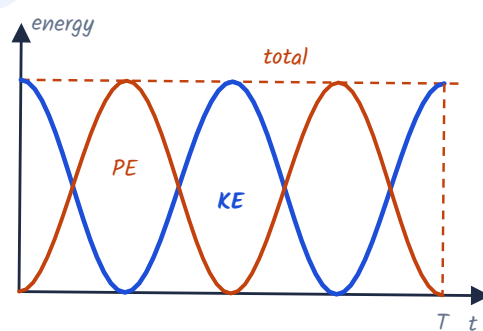
kinetic energy at displacement x
J

$$E_p = \frac{1}{2}m\omega^2x^2$$

potential energy at displacement x
J



Energy against displacement: KE greatest at the centre, PE at the extremes; the sum is constant.



Energy against time (starting at the centre): KE and PE swap twice per period.

Watch out!

- ! Energy goes as amplitude **squared**: halving x_0 gives a quarter of the energy.
- ! KE and PE vary with period **$T/2$** , not T .

17.3a Damped oscillations

Definitions

Damping: the loss of energy from an oscillating system, caused by resistive forces, resulting in a decrease in amplitude.

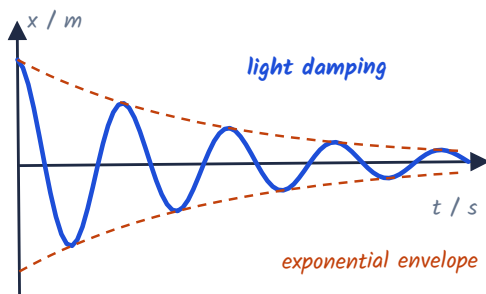
Light damping: the system oscillates with gradually decreasing amplitude.

Critical damping: the system returns to equilibrium in the shortest possible time without oscillating.

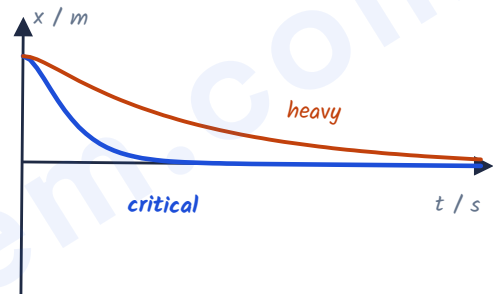
Heavy damping: the system returns slowly to equilibrium without oscillating.

Key points

- ✓ Resistive forces (friction, air resistance, viscosity) always act **against the velocity**, removing energy.
- ✓ Light damping: amplitude decays roughly **exponentially**; period almost unchanged.
- ✓ Critical damping is used in **car suspensions** and meter needles: fastest settling, no overshoot.
- ✓ Heavy damping: slower return than critical, still no oscillation.



Light damping: oscillations continue while the amplitude decays inside an exponential envelope.



Critical damping returns to equilibrium fastest with no overshoot; heavy damping creeps back slowly.

Watch out!

- ! Critical damping is the **minimum** damping that prevents oscillation, not "maximum damping".
- ! Damping reduces the amplitude, not (noticeably) the frequency, when it is light.

17.3b Forced oscillations and resonance

Definitions

Natural frequency (f_0): the frequency at which a system oscillates when displaced and left to oscillate freely.

Forced oscillation: an oscillation in which a periodic driving force makes the system oscillate at the driving frequency.

Resonance: the maximum amplitude of vibration when the driving frequency equals the natural frequency of the system.

Equations

$$f_{\text{driver}} = f_0$$

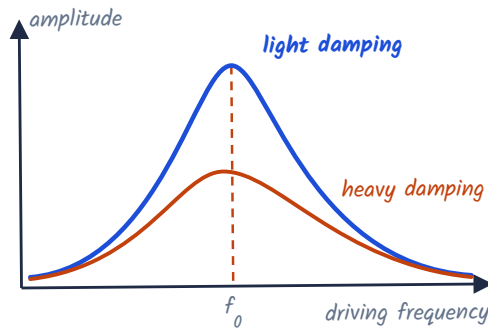
condition for resonance (light damping)

Hz

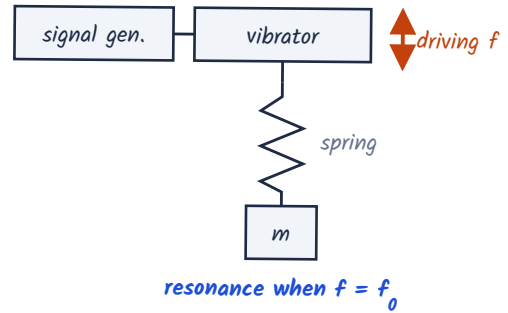
Key points

- ✓ A forced system oscillates at the **driving frequency**, not its natural frequency.
- ✓ At resonance energy is transferred from the driver **most efficiently**.

- ✓ Increasing damping: resonance peak **lower and broader**, and the peak moves to a slightly **lower frequency**.
- ✓ Useful resonance: microwave heating, MRI, tuning a radio circuit, musical instruments.
- ✓ Harmful resonance: bridges and buildings in wind or earthquakes, vibrating machinery; reduced by **damping**.



Resonance curves: amplitude peaks at the natural frequency f_0 ; heavier damping gives a lower, broader peak.



Forced oscillation: a vibration generator drives a mass-spring system; vary the driving frequency to find resonance.

Watch out!

- ! Resonance is when the **driving** frequency equals the **natural** frequency; the system always oscillates at the driving frequency.
- ! Damping does not remove resonance; it lowers and broadens the peak.