

Superposition

A-Level Physics 9702 · Topic 8 revision notes · AS

What happens when waves meet: stationary waves, diffraction, two-source interference, the double slit and the diffraction grating.

18 definitions to learn word for word, 12 equations in their own column, 12 diagrams and graphs. Read it, cover it, test yourself.

1. 8.1a Superposition and stationary waves
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Definitions (mark-scheme wording) Equations Exam traps

8.1a Superposition and stationary waves

Definitions

Principle of superposition: when two or more waves meet at a point, the resultant displacement is the sum of the displacements of the individual waves.

Stationary wave: the wave formed when two progressive waves of the same frequency (and similar amplitude) travelling in opposite directions superpose; no energy is transferred.

Node: a point on a stationary wave where the amplitude is always zero.

Antinode: a point on a stationary wave where the amplitude is a maximum.

Equations

$$\lambda = 2 \times (\text{node-to-node distance})$$

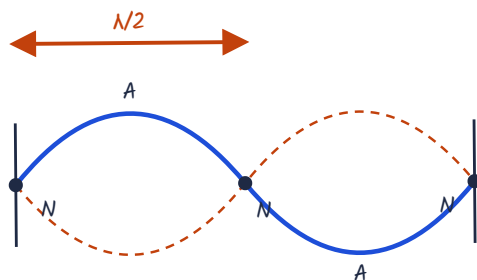
wavelength from the node spacing
m

$$v = f\lambda$$

wave speed of the component progressive waves
m s⁻¹

Key points

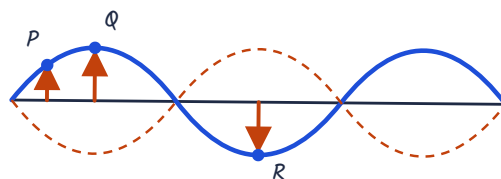
- ✓ Usually formed by a wave and its **reflection** travelling back along the same path.
- ✓ Adjacent nodes (or antinodes) are **$\lambda/2$ apart**; node to adjacent antinode is **$\lambda/4$** .
- ✓ All points between two adjacent nodes are **in phase**; points in adjacent loops are **180° out of phase**.
- ✓ Amplitude **varies with position** (zero at nodes); in a progressive wave every point has the same amplitude.
- ✓ A stationary wave **stores energy**; a progressive wave transfers energy.
- ✓ Wavelength of the stationary wave = wavelength of the two progressive waves that make it.



amplitude varies along the string

Stationary wave between fixed ends: solid and dashed shapes are half a period apart. Nodes N never move; antinodes A swing fully.

nodes every $\lambda/2$



P and Q: in phase

R: 180° out of phase with P

Phase in a stationary wave: P and Q (same loop) move in phase; R (next loop) is 180° out of phase with them.

Watch out!

- ! Adjacent nodes are $\lambda/2$ apart, not λ . Three nodes on a string span one whole wavelength.
- ! Points in the same loop are in phase even though their amplitudes differ.

8.16

Stationary waves on strings, in air columns and with microwaves

Definitions

Fundamental (first harmonic): the stationary wave of lowest frequency that can form on the string or in the air column.

Harmonic: a stationary wave mode whose frequency is a whole-number multiple of the fundamental frequency.

End correction: the small distance beyond the open end of a tube at which the antinode actually forms.

Key points

- ✓ String fixed at both ends: **nodes at both ends**. Fundamental $L = \lambda/2$; then $L = \lambda, 3\lambda/2 \dots$
- ✓ String apparatus: vibration generator, string over a pulley, hanging masses; change f until loops appear.
- ✓ Tube closed at one end: **node at the closed end, antinode at the open end**. Fundamental $L = \lambda/4$.
- ✓ Tube open at both ends: **antinodes at both ends**. Fundamental $L = \lambda/2$.
- ✓ Resonance tube: two successive resonance lengths differ by $\lambda/2$; subtracting cancels the end correction.
- ✓ Microwaves: transmitter faces a metal plate; move a probe along the line. Adjacent minima (nodes) are $\lambda/2$ apart.
- ✓ Measure across many nodes and divide, to reduce the percentage uncertainty.

Equations

$$L = n\lambda/2$$

string fixed at both ends or tube open at both ends ($n = 1, 2, 3 \dots$)

m

$$L = (2n - 1)\lambda/4$$

tube closed at one end; only odd harmonics

m

$$\lambda = 2(L_2 - L_1)$$

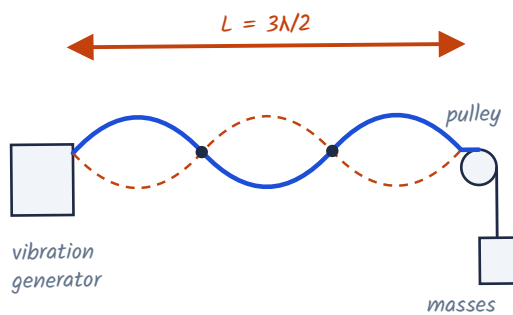
from two successive resonance lengths

m

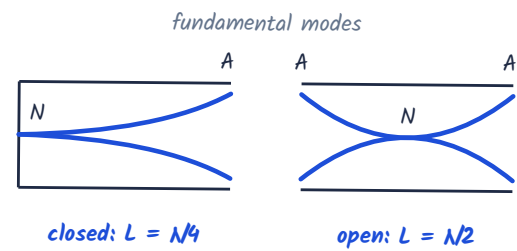
$$v = f\lambda$$

speed of sound or of the wave on the string

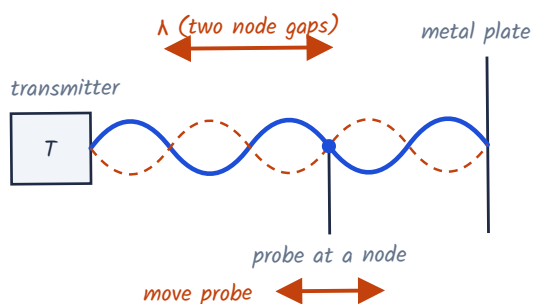
$m s^{-1}$



Stationary waves on a string: the vibration generator sets f ; the hanging masses set the tension. Here three loops, so $L = 3\lambda/2$.



Air columns (displacement envelopes): closed end = node, open end = antinode.



Microwaves reflected from a metal plate form a stationary wave; the probe finds nodes $\lambda/2$ apart.

Watch out!

- ! A closed tube has only **odd** harmonics ($L = \lambda/4, 3\lambda/4 \dots$); an open tube has all of them.
- ! The first resonance length alone gives $\lambda/4$ plus the end correction; use the difference of two lengths.

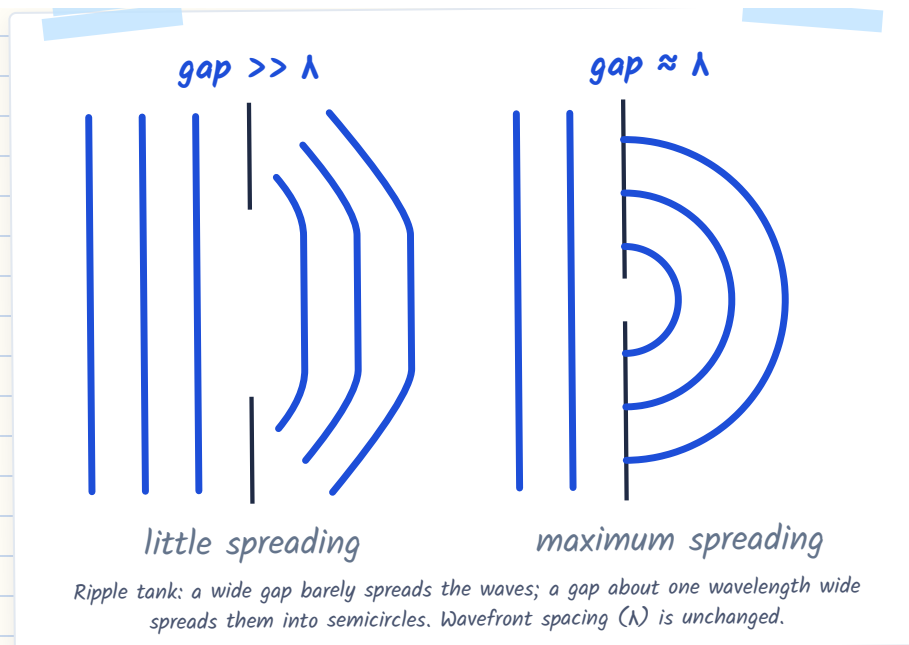
8.2 Diffraction

Definitions

Diffraction: the spreading of a wave as it passes through a gap or around the edge of an obstacle.

Key points

- ✓ Spreading is greatest when the **gap width is similar to the wavelength**.
- ✓ Gap much wider than λ : waves pass almost straight through, with slight spreading at the edges.
- ✓ **Wavelength, frequency and speed are unchanged** by diffraction; only the direction of travel spreads.
- ✓ Shown in a **ripple tank**: plane waves through a barrier gap become curved.
- ✓ Sound ($\lambda \sim 1 \text{ m}$) diffracts round doorways; light ($\lambda \sim 5 \times 10^{-7} \text{ m}$) needs a very narrow slit.
- ✓ Diffraction at each slit is what lets waves from two slits **overlap** and interfere.



Watch out!

- ! A narrower gap spreads the wave more, not less (down to a gap about λ wide).
- ! Do not draw a change in wavefront spacing after the gap: λ stays the same.

8.3a Interference and coherence

Definitions

Interference: the superposition of waves from two (or more) coherent sources to give regions of maximum and minimum amplitude.

Coherent: having a constant phase difference (so the waves have the same frequency).

Path difference: the difference in the distances travelled by two waves from their sources to a point.

Constructive interference: waves arrive in phase and superpose to give maximum amplitude.

Destructive interference: waves arrive in antiphase (180° out of phase) and superpose to give minimum amplitude.

Equations

$$\text{path difference} = n\lambda$$

constructive (maximum), in-phase sources
m

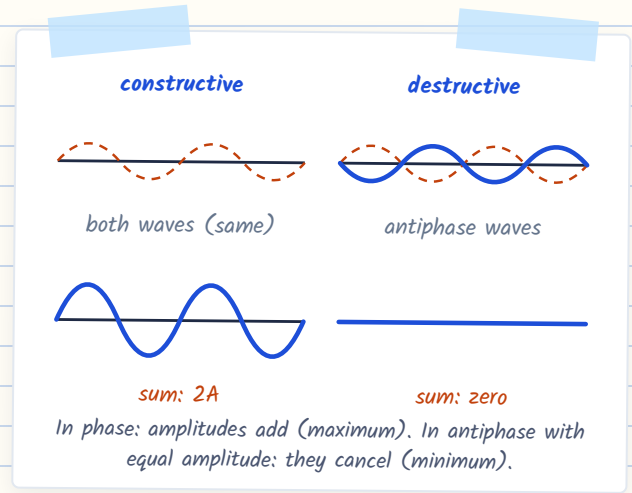
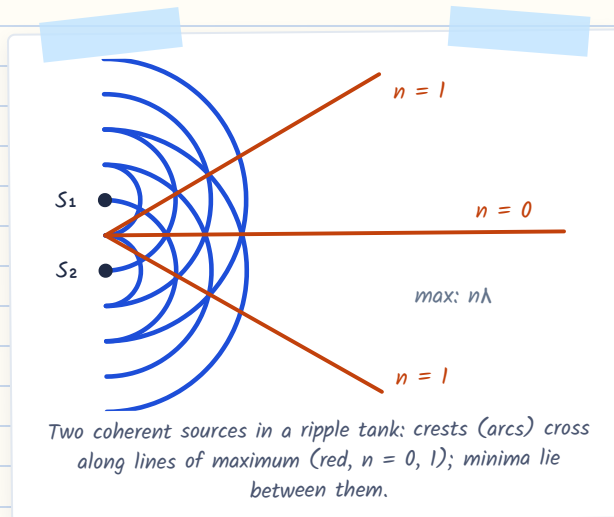
$$\text{path difference} = (n + \frac{1}{2})\lambda$$

destructive (minimum), in-phase sources
m

Key points

- ✓ A steady, observable pattern needs **coherent sources**; similar amplitudes give clear minima.
- ✓ For in-phase sources: **maximum** where path difference = $n\lambda$; **minimum** where it is $(n + \frac{1}{2})\lambda$.
- ✓ Equivalent phase differences: $0, 360^\circ \dots$ for a maximum; $180^\circ, 540^\circ \dots$ for a minimum.
- ✓ Complete cancellation only if the two amplitudes at that point are **equal**.
- ✓ Demonstrations: ripple tank with two dippers on one bar; two loudspeakers on one signal generator; microwaves through two slits.

- ✓ Two separate lamps are **not coherent** (random phase changes); use one laser, or one source and a double slit.



Watch out!

- ! Coherent means **constant** phase difference, not necessarily zero phase difference.
- ! If the sources are in antiphase the conditions swap: $n\lambda$ gives a minimum.

8.3b The double-slit experiment

Definitions

Fringe separation (x): the distance between the centres of adjacent bright (or adjacent dark) fringes.

Slit separation (a): the distance between the centres of the two slits.

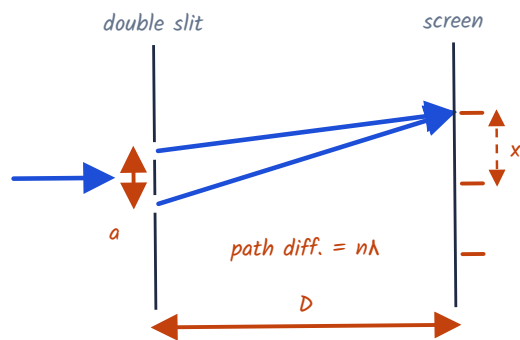
Equations

$$\lambda = ax / D$$

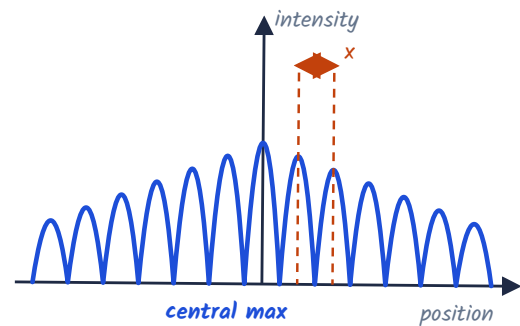
a = slit separation, x = fringe separation, D = slit-to-screen distance
 m

Key points

- ✓ Monochromatic light (e.g. a laser) through two narrow, close slits gives **equally spaced** bright and dark fringes.
- ✓ The slits act as **coherent sources**: both are lit by the same wavefront; diffraction makes their light overlap.
- ✓ Bright fringe: path difference $n\lambda$. Dark fringe: path difference $(n + \frac{1}{2})\lambda$.
- ✓ Fringe spacing x **increases** with λ and D , and **decreases** as a increases.
- ✓ Valid when **$a \ll D$** (small angles). Typical: $a \sim 0.5 \text{ mm}$, $D \sim 1-2 \text{ m}$, $x \sim \text{a few mm}$.
- ✓ Measure across **many fringes** and divide to reduce percentage uncertainty in x .
- ✓ White light: central fringe **white**; others are coloured spectra with violet nearest the centre, red furthest.



Double slit: light from the two slits reaches a bright fringe with path difference $n\lambda$; fringe spacing $x = \lambda D/a$.



Intensity across the screen: evenly spaced maxima, one fringe spacing x apart, fading slowly with distance from the centre.

Watch out!

- ! x is the spacing between **adjacent** fringes: 10 fringe spacings measured together means divide by 10, not 11.
- ! Keep units consistent: a and x often in mm, D in m. Convert all to metres.

8.4 The diffraction grating

Definitions

Diffraction grating: a plate with a large number of parallel, equally spaced, very narrow slits (lines).

Grating spacing (d): the distance between the centres of adjacent slits; $d = 1/N$, where N is the number of lines per metre.

Order (n): the number of wavelengths of path difference between waves from adjacent slits at that maximum.

Key points

- ✓ Many slits give maxima that are **very sharp and bright**, at large angles: more precise than the double slit.
- ✓ Zero order ($n = 0$) goes straight through; orders appear **symmetrically** on either side.
- ✓ Highest order: $\sin \theta \leq 1$, so $n_{\max} = d/\lambda$ **rounded down**.
- ✓ More lines per metre means smaller d , **larger angles** and fewer orders.
- ✓ Measure the angle between the two order- n maxima on either side and **halve it** to get θ .
- ✓ White light: zero order is white; each higher order is a spectrum, **violet deviated least, red most**; higher orders may overlap.

Equations

$$d \sin \theta = n\lambda$$

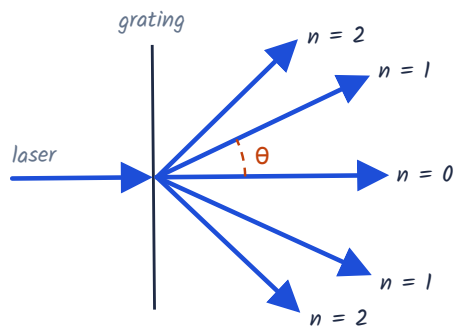
grating maxima; θ measured from the straight-through direction
 m

$$d = 1/N$$

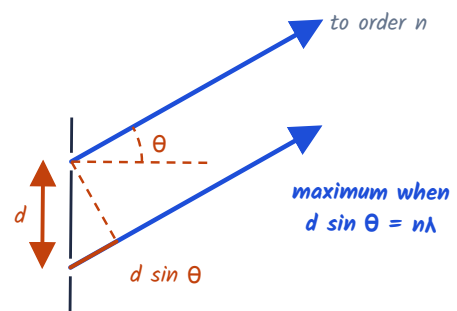
grating spacing from N lines per metre
 m

$$n_{\max} \leq d/\lambda$$

highest order visible ($\sin \theta = 1$ limit)
-



Grating orders: $n = 0$ straight through, symmetric orders either side at angles given by $d \sin \theta = n\lambda$.



Why $d \sin \theta = n\lambda$: waves from adjacent slits at angle θ differ in path by $d \sin \theta$; a maximum needs this to be a whole number of wavelengths.

Watch out!

- ! "500 lines per mm" means $N = 5 \times 10^5 \text{ m}^{-1}$, so $d = 2 \times 10^{-6} \text{ m}$. Convert to per metre before inverting.
- ! n_{max} is rounded **down**: $d/\lambda = 3.8$ means 3 orders each side (7 maxima in total).